XXII. *A Comparison of the late Imperial Standard Troy Pound weight with a Platina copy of the same, and with other standards of authority. Communicated by Professor* SCHUMACHER, *For. Memb. R.S., in a Letter to* F. BAILY, *Esq., V.P. and Treas. R.S.*

Received June 9,—Read June 16, 1836.

1. BEING desirous of obtaining an accurate copy of the English Imperial Standard Troy Pound, for an intended comparison of our weights therewith, I applied to the late Captain KATER, and he had the goodness to procure for me not only a copy made by Mr. BATE, exactly similar to those described in his paper *, but also a balance of Mr. ROBINSON, of the same dimension and construction as that used by himself in comparing the legal standard in the custody of the Clerk of the House of Commons. The copy of the Troy pound is of the same kind of brass as that used by Mr. BATE for the other copies sent by Captain KATER to different towns in Great Britain. It bears the stamp "Ty Pd 1824"; the same stamp, in fact, that was upon the pound No. 2.† which Captain KATER sent to Edinburgh. I shall designate this pound by the letter K. I received it, March 12, 1827, from the late Dr. YOUNG. He had noted upon the cover of the box, "Imperial Troy pound: found by Captain "KATER to exceed the standard a very little, not more than ·006 grain."

2. Fearing that this comparison (giving only *one* limit, for it is not said how much the difference is *below* 0·006 gr.) might not be made with that care which I thought necessary for the use I intended to make of the copy, I wrote again to Captain KATER, begging him to send me a second copy compared more carefully with the standard. He kindly undertook the task, ordered for me at Mr. ROBINSON's a second copy made of brass, together with divisions by halves, and sent it me in the summer of 1828, with the following notice:

"York Gate, Regent's Park, 18th July, 1828.

" .... I carefully compared the Troy pound which ROBINSON made for you with two separate " pounds of my own, the errors of which I had well determined. These pounds I designate the *old* " *pound* and the *unmarked pound*. I will copy for your satisfaction the comparisons at length.

Date.	Old Pound.	Prof. S.'s Pound.
1828.		
June 10.	4·5	3·1
	5·0	3·9
	5·7	3·8
June 11.	4·5	3·0
Mean...	4·92	3·45
Deduct..	1·83	3·09
	3·09	0·36

The *old pound* is heavier than the imperial pound ·0122 gr. = 1·83 div. (1·5 div. being = ·01 gr.)

Prof. S.'s pound too heavy 0·36 div. = 0·0024 gr.

* Philosophical Transactions, 1826.

† Ibid., p. 12.

Date.	Unmarked Pound.	Prof. S.'s Pound.
June 13.	3·3 4·6 4·8 4·0	3·3 3·5 3·8 3·6
Mean . . .	4·17 3·00	3·55 3·17
Deduct. .	7·17 4·00	0·38
	3·17	

N.B. ·02 gr. was placed in the opposite scale when the *unmarked pound* was counterpoised, and removed on replacing that pound with Prof. SCHUMACHER's pound.

The *unmarked pound* is heavier than the imperial pound ·0267 gr., or 4·0 divisions.

Prof. S.'s pound too heavy 0·38 div. = ·0025 gr.

"HENRY KATER."

The divisions to be deducted from the *unmarked* pound are, after an accurate calculation, 4·005 = 4·01. Of course ROBINSON's copy is, after the comparisons with the *unmarked* pound, too heavy 0·39 div. = 0·0026 gr., and by a mean of both pounds it is too heavy 0·0025 gr.

I call this new or second pound, made by ROBINSON, K^n, in order to distinguish it from the first pound, made by BATE, and compared for me by Captain KATER, which I have called K.

3. As soon as I received K^n, (Sept. 6, 1828,) I proceeded to compare it with K, and obtained the following comparisons, in which the divisions are already reduced to parts of the grain.

1828.	gr.	1828.	gr.
Sept. 7.	$K = K^n$ + 0·0191	Sept. 21.	$K = K^n$ + 0·0192
	+ 0·0204		+ 0·0209
	+ 0·0207	Sept. 22.	+ 0·0200
	+ 0·0182		+ 0·0216
	+ 0·0187		+ 0·0199
Sept. 21.	+ 0·0209		+ 0·0196
	+ 0·0193		+ 0·0207
	+ 0·0185		+ 0·0196
	+ 0·0188		+ 0·0216
	+ 0·0198		+ 0·0193
Mean of the three days $K = K^n + 0·0198$ gr. (20 comparisons.)			

4. This would give, assuming as zero Captain KATER's determination of K^n, (which he found too heavy 0·0025 gr.,) K too heavy 0·0223 gr. Therefore K, which, according to Captain KATER's first statement, should not exceed the standard more than 0·006 gr., exceeded it 0·022 gr. This discordance being far too considerable for the powers of ROBINSON's balances, and not suggesting such a difference of specific gravity between the two kinds of brass (BATE's and ROBINSON's) that might explain it, I sent K back to Captain KATER, with a request that he would compare it once more with his two pounds; explaining at the same time the cause of the additional trouble

I gave him. He had the goodness to comply with my wish. I received an answer, (out of which I extract the passage relating to the comparisons in 1827 and 1829,) with the following notice, which I subjoin entire.

"York Gate, Regent's Park, 31st May, 1829.

" . . . . I have the pleasure at length to send you the comparisons of your Troy pound with two " of mine. I am totally at a loss to conceive how this pound can have undergone the alteration you " mention; for as to an error of even the hundredth of a grain in the mean of two or three compari- " sons, it appears to me to be impossible. When I formerly tried it, it was merely with a view to see " that Mr. Bate had not made any great error, and I therefore made only two or three comparisons. " It is useless, however, to speculate further upon this.

" The comparisons I now send you have been made with all the care I could bestow on them; " and I think, taking the mean value of this pound and the one by Robinson I before examined, you " will be *very* near the truth.

" Henry Kater.

" Comparisons of Professor Schumacher's Troy Pound, made by Bate, with Captain Kater's two Troy Pounds.

Date.	Capt. K.'s Old Pound.	Prof. S.'s Pound.	Capt. K.'s Unmarked Pound.
1829.	Div.	Div.	Div.
Feb. 18.	0·6	1·5	1·9
19.	0·6	1·6	1·5
	0·6	1·9	1·6
	0·5	1·6	1·6
20.	0·4	1·1	1·3
	0·2	1·1	1·3
	0·9	2·9	2·7
21.	0·8	2·8	2·6
	1·1	3·4	3·7
22.	1·0	2·7	3·0
	1·1	3·3	3·0
	1·1	2·5	2·7
23.	1·2	2·9	2·4
	0·5	2·9	2·7
24.	1·1	3·3	2·8
26.	1·7	3·5	3·3
27.	1·7	3·8	3·2
Mar. 9.	0·6	1·8	1·2
	0·4	1·9	1·2
10.	0·3	2·2	2·1
Mean ..	0·79	2·44	2·29
	2·44		2·44
	+1·65		+0·15

One hundredth of a grain was found to be = 1·38 div.
Hence 1 div. = 0·00725 gr. (See below.)

Zero changed.

Professor Schumacher's pound heavier than the old pound 1·65 div.	= ·0119 gr.
Old pound heavier than the imperial pound	·0122
Professor Schumacher's pound heavier than the imperial pound	·0241
Professor Schumacher's pound heavier than the unmarked pound 0·15 div.	= ·0011
Unmarked pound heavier than the imperial pound......................	·0267
Prof. S.'s pound — By unmarked pound ..	+ ·0278
Prof. S.'s pound — By old pound.......	+ ·0241
Mean	+ ·0259

" The above value of the divisions of the balance was determined by means of six different weights " of ·03 gr. each, and three of ·02 gr. each, one pound being in each scale.

Div.	Diff.
0·8	
	4·7
5·5	
	4·2
1·3	
	4·3
5·6	
	4·0
1·6	
	4·0
5·6	
	4·1
1·5	
	4·1
5·6	
	3·8
1·8	
	4·2
6·0	
	4·6
1·4	
	3·9
5·3	
Mean	4·17

$\frac{1}{3}$ = 1·39 = ·01 gr.

Div.	Diff.
1·9	
	2·8
4·7	
	2·6
2·1	
	2·9
5·0	
	3·1
1·9	
	2·2
4·1	
Mean =	2·72

$\frac{1}{2}$ = 1·36 = ·01 gr."

5. Captain KATER's last result, by which he found K too heavy 0·0259 gr., agrees with that found by me, by ROBINSON's pound, within 0·0036 gr., but differs from its first result 0·0299 gr. He seems inclined to explain it by an increase of weight in the mean time, and this was also my opinion when I sent the pound back to him. It had not then the brilliancy of polish which it had when it arrived; and I thought oxydation might have increased the weight; but when I had it back in the autumn of 1829, I compared it again with the weights inclosed in ROBINSON's balance, with which it had been compared upon its arrival in 1827, and there was no sensible difference from the first comparison *. It is therefore more probable that the first comparison made by Captain KATER with his own weights, being only intended to see that Mr. BATE *had not made any great error*, were not made with that care with which he compared it afterwards in 1829.

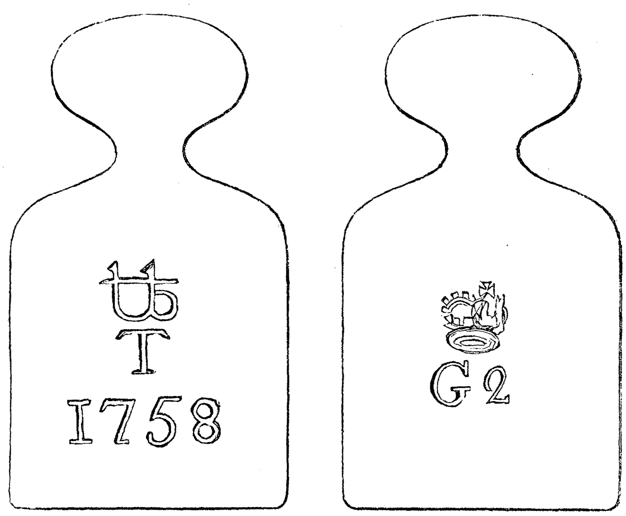
6. But before I got Captain KATER's answer, (of May 31, 1829,) I considered that a copy from two copies would hardly answer my purpose, and that, to obtain the accuracy at which I aimed, I should have a copy from the *original*, and that that copy, in order to preserve its accuracy, must not be made of a metal liable to oxydation, but of platina. I wanted at the same time more numerous comparisons than I could with any propriety charge my English friends with; and resolved to send one of my assistants (Captain NEHUS, of the Royal Danish Engineers) to London, in order to make them. A platina pound was therefore ordered of Mr. ROBINSON.

Our Government applied to the English Government to obtain for Captain NEHUS

* The brass weights of ROBINSON with which it was compared were of 5000 gr., 400 gr., 300 gr., (the rest were of platina,) and had a larger surface than BATE's pound. It is possible that the three brass weights of ROBINSON may in the mean time also have increased in weight by oxydation; but this increase ought to have been just equal to that of BATE's pound, as the comparisons in 1827 and 1829 gave the same result. Now it is not very probable that ROBINSON's weights should have been subjected to a smaller oxydation, and just in the inverse ratio of the surfaces smaller than BATE's pound. It is more probable that both weights have not in those two years suffered from oxydation.

free access to the Imperial Standard Troy Pound, and the permission to compare it with my copies; but this application proved unnecessary, because, before it could be made, the President of the Royal Society, Davies Gilbert, Esq., had the kindness to intercede with the Speaker of the House of Commons in Captain Nehus's behalf, and obtained permission to bring the Imperial Standard Troy Pound, upon his own responsibility, to the Apartments of the Royal Society in Somerset House, where it was deposited in the Council-Room: which place was assigned to Captain Nehus for his comparisons. Mr. Gilbert's kindness extended still further. The balance intended for these comparisons, and ordered of Mr. Robinson, not being ready when Captain Nehus arrived, he permitted him to use Ramsden's balance belonging to the Royal Society until Robinson's could be obtained. Captain Nehus experienced likewise during the course of his comparisons marks of uninterrupted kindness and attention from Mr. Hudson, then Assistant Secretary of the Royal Society.

7. As soon as the Imperial Standard Troy Pound was brought to Somerset House, Captain Nehus's first care was to make an accurate drawing of its shape and marks, measuring all its dimensions with the greatest care. The annexed drawing represents this pound in its actual dimensions; and is now, since the original has been destroyed by the calamitous fire that consumed the two Houses of Parliament in November 1834, the only thing remaining which can preserve an idea of it.



I give now Captain Nehus's comparisons, first made with Ramsden's balance, afterwards with Robinson's. The Imperial Standard Troy Pound is designated by U (Unit), my Platina copy by S.P. The difference of weight is expressed in parts of the scale, the value of which will be shown in the sequel (see pages 465 and 466.). The thermometers L and R were suspended in the box of the balance; L at left, R at right hand.

With Ramsden's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
1.	S.^{p} = U − 1·375 pt.	29·726	69°·8	67°·0	67°·7	5.	S.^{p} = U − 1·900 pt.		°	°	°
2.	S.^{p} = U − 0·500					6.	S.^{p} = U − 1·250				
3.	S.^{p} = U − 0·750					7.	S.^{p} = U − 0·875				
4.	S.^{p} = U − 0·350					8.	S.^{p} = U − 1·025	29·744	68·7	67·1	67·8

1829, June 21. Mean of 8 ...... S.^{p} = U − 1·003 pt. Barom. 29·735 Att. Th. 69°·25 Th. L. 67°·05 Th. R. 67°·75

9.	S.^{p} = U − 2·008	29·870	71·1	66·9	67·0	17.	S.^{p} = U − 0·675				
10.	S.^{p} = U + 0·033	29·883	69·3			18.	S.^{p} = U − 1·325				
11.	S.^{p} = U − 1·700	......	......	67·0	67·4	19.	S.^{p} = U − 0·800	......	......	67·3	67·9
12.	S.^{p} = U − 0·925					20.	S.^{p} = U − 1·617				
13.	S.^{p} = U − 1·850					21.	S.^{p} = U − 1·425				
14.	S.^{p} = U − 3·000					22.	S.^{p} = U − 1·250				
15.	S.^{p} = U − 0·275					23.	S.^{p} = U − 1·108				
16.	S.^{p} = U − 0·433					24.	S.^{p} = U − 0·250	29·904	68·4	67·2	68·0

1829, June 23. Mean of 16 ...... S.^{p} = U − 1·163 pt. Barom. 29·886 Att. Th. 69°·6 Th. L. 67°·1 Th. R. 67°·57

25.	S.^{p} = U − 1·050	29·992	75·9	67·8	67·8	33.	S.^{p} = U − 2·558				
26.	S.^{p} = U − 0·900					34.	S.^{p} = U − 1·950	......	......	68·5	68·0
27.	S.^{p} = U − 0·350					35.	S.^{p} = U + 0·200				
28.	S.^{p} = U − 1·350					36.	S.^{p} = U − 0·450				
29.	S.^{p} = U + 0·400					37.	S.^{p} = U − 1·200				
30.	S.^{p} = U − 1·350					38.	S.^{p} = U − 1·950				
31.	S.^{p} = U − 0·650					39.	S.^{p} = U − 0·175				
32.	S.^{p} = U − 1·550					40.	S.^{p} = U − 0·625				

1829, June 24. Mean of 16 ...... S.^{p} = U − 0·969 pt. Barom. 29·992 Att. Th. 75°·9 Th. L. 68°·15 Th. R. 67°·9

41.	S.^{p} = U − 2·375					46.	S.^{p} = U − 1·950				
42.	S.^{p} = U − 2·500					47.	S.^{p} = U − 0·150				
43.	S.^{p} = U − 0·650					48.	S.^{p} = U − 1·100				
44.	S.^{p} = U − 1·300					49.	S.^{p} = U − 0·150				
45.	S.^{p} = U + 0·400					50.	S.^{p} = U − 0·150	30·018	69·9	68·2	68·0

1829, June 24. Mean of 10 ...... S.^{p} = U − 0·992 pt. Barom. 30·018 Att. Th. 69°·9 Th. L. 68°·2 Th. R. 68°·0

51.	S.^{p} = U − 1·65	29·375	70·4	68·8	68·8	56.	S.^{p} = U − 2·60				
52.	S.^{p} = U − 2·25					57.	S.^{p} = U − 2·90				
53.	S.^{p} = U − 0·55					58.	S.^{p} = U − 1·50				
54.	S.^{p} = U − 2·10					59.	S.^{p} = U − 1·30				
55.	S.^{p} = U − 3·10					60.	S.^{p} = U − 2·00	29·388	70·6	69·2	69·6

1829, June 28. Mean of 10 ...... S.^{p} = U − 1·995 pt. Barom. 29·381 Att. Th. 70°·5 Th. L. 69°·0 Th. R. 69°·2

61.	S.^{p} = U − 2·60	29·660	65·5	66·5	66·8	66.	S.^{p} = U − 0·55	......	......	66·9	67·2
62.	S.^{p} = U − 1·85					67.	S.^{p} = U 0·00				
63.	S.^{p} = U − 1·40					68.	S.^{p} = U + 0·45				
64.	S.^{p} = U − 2·85					69.	S.^{p} = U − 2·80				
65.	S.^{p} = U − 1·20					70.	S.^{p} = U − 1·30	29·684	65·9		

1829, June 29. Mean of 10 ...... S.^{p} = U − 1·410 pt. Barom. 29·672 Th. Att. 65°·7 Th. L. 66°·7 Th. R. 67°·0

71.	S.^{p} = U − 2·65	29·583	65·0	65·2	65·2	76.	S.^{p} = U − 2·90	......	......	67·0	67·8
72.	S.^{p} = U − 2·45					77.	S.^{p} = U − 2·80				
73.	S.^{p} = U − 0·65					78.	S.^{p} = U − 2·55				
74.	S.^{p} = U − 2·55					79.	S.^{p} = U − 1·90				
75.	S.^{p} = U − 2·60					80.	S.^{p} = U − 1·15	29·494	65·5		

1829, July 1. Mean of 10 ...... S.^{p} = U − 2·22 pt. Barom. 29·538 Att. Th. 65°·25 Th. L. 66°·1 Th. R. 66°·5

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
81.	S.^{p} = U − 1·80 pt.	29·554	63°·2	65°·0	65°·5	86.	S.^{p} = U − 1·85 pt.		°	°	°
82.	S.^{p} = U − 1·50					87.	S.^{p} = U − 1·15				
83.	S.^{p} = U − 0·25					88.	S.^{p} = U − 1·50	....	....	65·4	65·8
84.	S.^{p} = U − 1·25					89.	S.^{p} = U − 0·70				
85.	S.^{p} = U − 2·25					90.	S.^{p} = U − 1·90	29·532	63·7		

1829, July 4. Mean of 10 ...... S.^{p} = U − 1·415 pt. Barom. 29·543 Att. Th. 63°·45 Th. L. 65°·2 Th. R. 65°·65

91.	S.^{p} = U − 1·30	29·530	63·2	64·0	64·1	96.	S.^{p} = U − 1·95	....	....	65·2	65·4
92.	S.^{p} = U − 1·80					97.	S.^{p} = U − 2·25				
93.	S.^{p} = U − 1·35					98.	S.^{p} = U − 1·35				
94.	S.^{p} = U − 1·10					99.	S.^{p} = U − 1·95				
95.	S.^{p} = U − 2·35					100.	S.^{p} = U − 3·30				

1829, July 5. Mean of 10 ...... S.^{p} = U − 1·870 pt. Barom. 29·530 Att. Th. 63°·2 Th. L. 64°·6 Th. R. 64°·75

With Robinson's Balance.

1.	S.^{p} = U − 1·60					6.	S.^{p} = U − 1·85				
2.	S.^{p} = U − 1·35					7.	S.^{p} = U − 2·15	....	....	69·4	69·8
3.	S.^{p} = U − 1·65	....	....	69·5	69·8	8.	S.^{p} = U − 0·95				
4.	S.^{p} = U − 1·70					9.	S.^{p} = U − 1·30				
5.	S.^{p} = U − 1·85					10.	S.^{p} = U − 1·70	29·462	68·8		

1829, June 28. Mean of 10 ...... S.^{p} = U − 1·610 pt. Barom. 29·462 Att. Th. 68°·8 Th. L. 69°·45 Th. R. 69°·8

11.	S.^{p} = U − 1·00	29·636	65·2	66·0	66·2	16.	S.^{p} = U − 0·25				
12.	S.^{p} = U − 0·55					17.	S.^{p} = U − 0·25				
13.	S.^{p} = U + 0·30					18.	S.^{p} = U + 0·40				
14.	S.^{p} = U − 0·40					19.	S.^{p} = U − 2·00				
15.	S.^{p} = U − 1·65	....	....	66·8	67·0	20.	S.^{p} = U − 2·15	29·660	65·5	67·1	67·3

1829, June 29. Mean of 10 ...... S.^{p} = U − 0·755 pt. Barom. 29·648 Att. Th. 65°·35 Th. L. 66°·6 Th. R. 66°·8

21.	S.^{p} = U − 2·75	29·460	65·5	65·8	66·0	26.	S.^{p} = U − 1·35				
22.	S.^{p} = U − 1·60					27.	S.^{p} = U − 1·40				
23.	S.^{p} = U − 2·65	....	....	66·8	67·0	28.	S.^{p} = U − 1·30				
24.	S.^{p} = U − 2·65					29.	S.^{p} = U − 1·65	....	....	67·0	67·1
25.	S.^{p} = U − 2·00					30.	S.^{p} = U − 1·30	29·406	66·0		

1829, July 1. Mean of 10 ...... S.^{p} = U − 1·865 pt. Barom. 29·433 Att. Th. 65°·75 Th. L. 66°·5 Th. R. 66°·7

31.	S.^{p} = U − 1·65	29·532	63·7	65·0	65·2	41.	S.^{p} = U − 0·90	29·510	64·2		
32.	S.^{p} = U − 0·90					42.	S.^{p} = U − 1·30				
33.	S.^{p} = U − 1·00					43.	S.^{p} = U − 2·25				
34.	S.^{p} = U − 1·45					44.	S.^{p} = U − 1·50				
35.	S.^{p} = U − 1·20					45.	S.^{p} = U − 0·75				
36.	S.^{p} = U − 1·45					46.	S.^{p} = U − 0·80				
37.	S.^{p} = U − 1·45					47.	S.^{p} = U − 1·45				
38.	S.^{p} = U − 0·75					48.	S.^{p} = U − 0·95				
39.	S.^{p} = U − 1·45					49.	S.^{p} = U − 1·50				
40.	S.^{p} = U − 0·65	....	....	65·1	65·3	50.	S.^{p} = U − 1·60	29·513	64·2	65·2	65·4

1829, July 4. Mean of 20 ...... S.^{p} = U − 1·247 pt. Barom. 29·518 Att. Th. 64°·0 Th. L. 65°·1 Th. R. 65°·3

51.	S.^{p} = U − 1·65	29·596	63·8	64·8	65·0	61.	S.^{p} = U − 1·50				
52.	S.^{p} = U − 0·65					62.	S.^{p} = U − 0·95				
53.	S.^{p} = U − 0·45					63.	S.^{p} = U − 1·25				
54.	S.^{p} = U + 0·10					64.	S.^{p} = U − 0·80				
55.	S.^{p} = U − 0·70					65.	S.^{p} = U − 1·40				
56.	S.^{p} = U − 1·15					66.	S.^{p} = U − 1·45				
57.	S.^{p} = U − 1·50					67.	S.^{p} = U − 1·25				
58.	S.^{p} = U − 0·75					68.	S.^{p} = U − 0·50				
59.	S.^{p} = U − 0·75	....	....	65·2	65·4	69.	S.^{p} = U − 1·15				
60.	S.^{p} = U − 0·60					70.	S.^{p} = U − 0·80				

1829, July 5. Mean of 20 ...... S.^{p} = U − 0·9575 pt. Barom. 29·596 Att. Th. 63°·8 Th. L. 65°·0 Th. R. 65°·2

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	pt. gr.		°	°	°		pt. gr.		°	°	°
71.	S.^{p} = U + 0·15 − 0·01	29·690	64·4	65·0	65·2	86.	S.^{p} = U − 0·10 − 0·01				
72.	S.^{p} = U − 0·50 − 0·01					87.	S.^{p} = U + 0·35 − 0·01				
73.	S.^{p} = U + 0·45 − 0·01					88.	S.^{p} = U + 0·05 − 0·01				
74.	S.^{p} = U − 0·95 − 0·01					89.	S.^{p} = U + 0·20 − 0·01				
75.	S.^{p} = U + 0·50 − 0·01					90.	S.^{p} = U + 0·70 − 0·01				
76.	S.^{p} = U + 0·10 − 0·01					91.	S.^{p} = U + 0·30 − 0·01	....	....	65·8	66·0
77.	S.^{p} = U 0·00 − 0·01	....	....	65·8	66·0	92.	S.^{p} = U + 0·05 − 0·01				
78.	S.^{p} = U − 0·45 − 0·01					93.	S.^{p} = U + 0·10 − 0·01				
79.	S.^{p} = U + 0·05 − 0·01					94.	S.^{p} = U − 0·10 − 0·01				
80.	S.^{p} = U − 0·20 − 0·01					95.	S.^{p} = U + 0·90 − 0·01				
81.	S.^{p} = U − 0·10 − 0·01					96.	S.^{p} = U + 0·30 − 0·01				
82.	S.^{p} = U 0·00 − 0·01					97.	S.^{p} = U − 0·25 − 0·01				
83.	S.^{p} = U + 0·50 − 0·01					98.	S.^{p} = U + 0·15 − 0·01				
84.	S.^{p} = U + 0·10 − 0·01					99.	S.^{p} = U + 0·10 − 0·01				
85.	S.^{p} = U − 0·10 − 0·01					100.	S.^{p} = U 0·00 − 0·01	29·750	65·0	65·9	66·1

1829, July 9. Mean of 30 ...... S.^{p} = U + 0·070 pt. − 0·01 gr. Barom. 29·720 Att. Th. 64°·7 Th. L. 65°·57 Th. R. 65°·77

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	pt.						pt.				
101.	S.^{p} = U − 0·05	30·188	63·2	64·0	64·2	111.	S.^{p} = U + 0·05	....	....	64·8	65·0
102.	S.^{p} = U + 0·20					112.	S.^{p} = U + 0·35				
103.	S.^{p} = U + 0·10					113.	S.^{p} = U − 0·05				
104.	S.^{p} = U + 0·05					114.	S.^{p} = U + 0·25				
105.	S.^{p} = U + 0·30					115.	S.^{p} = U − 0·15				
106.	S.^{p} = U − 0·05	....	....	64·7	64·9	116.	S.^{p} = U − 0·05				
107.	S.^{p} = U + 0·25					117.	S.^{p} = U + 0·10				
108.	S.^{p} = U − 0·15					118.	S.^{p} = U + 0·35				
109.	S.^{p} = U − 0·30					119.	S.^{p} = U + 0·30				
110.	S.^{p} = U − 0·45					120.	S.^{p} = U − 0·20	30·170	64·0		

1829, August 1. Mean of 20 ...... S.^{p} = U + 0·0425 pt. Barom. 30·179 Att. Th. 63°·6 Th. L. 64°·5 Th. R. 64°·7

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
121.	S.^{p} = U 0·00	30·188	64·6	66·0	66·0	131.	S.^{p} = U − 0·20				
122.	S.^{p} = U − 0·15					132.	S.^{p} = U − 0·40				
123.	S.^{p} = U − 0·10					133.	S.^{p} = U − 0·20				
124.	S.^{p} = U − 0·15					134.	S.^{p} = U − 0·40				
125.	S.^{p} = U 0·00					135.	S.^{p} = U 0·00				
126.	S.^{p} = U − 0·35					136.	S.^{p} = U − 0·30	......	......	65·8	66·0
127.	S.^{p} = U − 0·40					137.	S.^{p} = U − 0·30				
128.	S.^{p} = U − 0·05	......	......	66·0	66·0	138.	S.^{p} = U − 0·90				
129.	S.^{p} = U − 0·05					139.	S.^{p} = U − 0·65				
130.	S.^{p} = U − 0·45					140.	S.^{p} = U − 0·40	30·184	64·6		

1829, August 2. Mean of 20 ...... S.^{p} = U − 0·2725 pt. Barom. 30·186 Att. Th. 64°·6 Th. L. 65°·93 Th. R. 66°·0

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	pt. gr.										
141.	S.^{p} = U + 0·50 − 0·01	30·052	64·5	65·2	65·4	156.	S.^{p} = U − 0·50				
142.	S.^{p} = U + 0·60 − 0·01					157.	S.^{p} = U − 0·15				
143.	S.^{p} = U + 0·55 − 0·01					158.	S.^{p} = U − 0·60				
144.	S.^{p} = U + 0·15 − 0·01					159.	S.^{p} = U − 0·30				
145.	S.^{p} = U + 0·70 − 0·01					160.	S.^{p} = U − 0·25				
146.	S.^{p} = U + 0·15 − 0·01					161.	S.^{p} = U − 0·20	29·916	64·8	65·8	66·0
147.	S.^{p} = U + 0·60 − 0·01	......	......	65·8	66·0	162.	S.^{p} = U − 0·10				
148.	S.^{p} = U + 0·45 − 0·01					163.	S.^{p} = U − 0·25				
149.	S.^{p} = U − 0·60					164.	S.^{p} = U − 0·65				
150.	S.^{p} = U − 0·95					165.	S.^{p} = U − 0·35				
151.	S.^{p} = U − 0·65					166.	S.^{p} = U − 0·70				
152.	S.^{p} = U − 0·20					167.	S.^{p} = U − 0·10				
153.	S.^{p} = U 0·00					168.	S.^{p} = U − 0·95				
154.	S.^{p} = U − 0·75					169.	S.^{p} = U − 0·55				
155.	S.^{p} = U − 0·20	......	......	65·0	65·2	170.	S.^{p} = U − 0·70	29·882	65·0	66·0	66·1

1829, August 3. Mean of 30 ...... S.^{p} = U − $\frac{6·00 \text{ pt.} - 0·08 \text{ gr.}}{30}$ Barom. 29·950 Att. Th. 64°·77 Th. L. 65°·56 Th. R. 65°·74

= U − 0·20 pt. − 0·002667 gr.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	gr. pt.		°	°	°		gr. pt.		°	°	°
171.	S.^{p} = U − 0·01 + 0·45	29·736	63·9	64·9	65·2	186.	S.^{p} = U − 0·01 + 0·10				
172.	S.^{p} = U − 0·01 + 0·35					187.	S.^{p} = U − 0·01 + 0·55				
173.	S.^{p} = U − 0·01 + 0·50					188.	S.^{p} = U − 0·01 + 0·20				
174.	S.^{p} = U − 0·01 + 0·40					189.	S.^{p} = U − 0·01 + 0·40				
175.	S.^{p} = U − 0·01 + 0·80					190.	S.^{p} = U − 0·01 + 0·60				
176.	S.^{p} = U − 0·01 − 0·25	......	......	65·0	65·2	191.	S.^{p} = U − 0·01 + 0·55				
177.	S.^{p} = U − 0·01 + 0·70					192.	S.^{p} = U − 0·01 + 0·90	......	......	65·0	65·3
178.	S.^{p} = U − 0·01 + 0·35					193.	S.^{p} = U − 0·01 + 0·35				
179.	S.^{p} = U − 0·01 + 0·75					194.	S.^{p} = U − 0·01 + 0·35				
180.	S.^{p} = U − 0·01 + 0·20					195.	S.^{p} = U − 0·01 + 0·65				
181.	S.^{p} = U − 0·01 + 0·20					196.	S.^{p} = U − 0·01 + 0·15				
182.	S.^{p} = U − 0·01 + 0·20					197.	S.^{p} = U − 0·01 + 1·00				
183.	S.^{p} = U − 0·01 + 1·10					198.	S.^{p} = U − 0·01 − 0·05				
184.	S.^{p} = U − 0·01 + 0·55	......	......	65·1	65·4	199.	S.^{p} = U − 0·01 + 0·75				
185.	S.^{p} = U − 0·01 + 0·55					200.	S.^{p} = U − 0·01 + 0·70	29·710	63·8		

1829, August 4. Mean of 30 ...... S.^{p} = U − 0·01 gr. + 0·4683 pt.; Barom. 29·723; Th. Att. 63°·85; Th. L. 65°·0; Th. R. 65°·27

8. It remains now, for the purpose of obtaining the results of these comparisons, to determine the values of the parts or divisions of the graduated arc of the balance under the pressure of one troy pound in each scale, for Ramsden's and Robinson's balances, to reduce the indications of the barometer to absolute heights at 32° temperature, and to find the errors (if any) of the thermometers employed.

9. For determining the values of the divisions or parts of the graduated arc of Ramsden's balance, the following observations were made, one troy pound being in each scale.

	grs. pts.		grs. pts.		grs. pts.		grs. pts.
June 21.	0·04 = 4·575	June 21.	0·04 = 4·620	June 21.	0·04 = 4·570	June 22.	0·04 = 5·750
21.	0·04 = 6·250	21.	0·04 = 4·550	22.	0·04 = 4·900	22.	0·04 = 4·375
Mean of the two days ...... 0·32 gr. = 39·590 parts, or 1 part = 0·00808 gr.							
The centre of gravity of the balance was altered.							
June 23.	0·04 = 6·738	June 24.	0·04 = 5·375	June 28.	0·04 = 4·950	July 4.	0·04 = 6·880
23.	0·04 = 6·033	24.	0·04 = 5·700	29.	0·04 = 6·550	4.	0·04 = 6·825
23.	0·04 = 6·575	24.	0·04 = 5·500	29.	0·04 = 5·325	5.	0·02 = 3·875
23.	0·04 = 7·279	24.	0·04 = 5·425	July 1.	0·04 = 6·850	5.	0·02 = 2·150
24.	0·04 = 6·138	28.	0·04 = 5·875	1.	0·04 = 5·525		
Mean of the seven days ...... 0·72 gr. = 109·568 parts, or 1 part = 0·00657 gr.							
The centre of gravity of the balance was altered.							
August 2.	0·02 = 2·800	August 2.	0·02 = 1·800	August 4.	0·02 = 1·300	August 4.	0·02 = 1·300
2.	0·02 = 2·200	4.	0·02 = 1·950	4.	0·02 = 1·500		
2.	0·02 = 1·100	4.	0·02 = 2·100	4.	0·02 = 1·400		
Mean of the two days ...... 0·20 gr. = 17·450 parts, or 1 part = 0·01146 gr.							

Of course we must employ, for the reduction of the comparisons, the following values of the parts:

June 21, 22.	1 part = 0·00808 gr.	Ramsden's balance.
June 23, 24, 28, 29, July 1, 4, 5.	1 part = 0·00657	
August 2, 4.	1 part = 0·01146	

The centre of gravity of Robinson's balance was never altered during the course of the observations, so that upon that account there seems no objection to employ the totality of the determinations of the values of the parts of the graduated scale for reducing all the observations made with it. All these determinations were made with one troy pound in each scale.

	gr. pts.		gr. pts.		gr. pts.		gr. pts.
June 28.	0·02 = 2·125	July 5.	0·02 = 1·800	July 11.	0·02 = 1·550	July 14.	0·02 = 1·550
28.	0·02 = 2·050	5.	0·02 = 1·825	11.	0·02 = 1·650	15.	0·02 = 1·500
29.	0·02 = 1·975	7.	0·02 = 1·575	12.	0·02 = 1·800	15.	0·02 = 1·500
29.	0·02 = 1·850	7.	0·02 = 1·650	12.	0·02 = 1·700	15.	0·02 = 1·600
July 1.	0·02 = 1·575	7.	0·02 = 1·750	12.	0·02 = 1·750	15.	0·02 = 2·050
1.	0·02 = 1·450	9.	0·02 = 1·725	12.	0·02 = 1·650	16.	0·02 = 1·700
4.	0·02 = 1·800	9.	0·02 = 1·725	12.	0·02 = 1·700	16.	0·02 = 1·500
4.	0·02 = 1·935	9.	0·02 = 1·650	14.	0·02 = 1·650	16.	0·02 = 1·600
4.	0·02 = 1·675	9.	0·02 = 1·775	14.	0·02 = 1·650	16.	0·02 = 1·900
5.	0·02 = 1·575	9.	0·02 = 1·750	14.	0·02 = 1·500		
Mean from June 28 to July 16 ...... 0·78 gr. = 66·735 parts, or 1 part = 0·01169 gr.							
July 30.	0·02 = 2·100	August 2.	0·02 = 1·875	August 4.	0·02 = 1·875	August 6.	0·02 = 1·900
August 1.	0·02 = 2·025	2.	0·02 = 1·950	4.	0·02 = 1·825	6.	0·02 = 1·925
1.	0·02 = 2·000	2.	0·02 = 1·950	5.	0·02 = 1·950		
2.	0·02 = 1·875	3.	0·02 = 2·075	5.	0·02 = 1·875		
Mean from July 30 to August 6 ...... 0·28 gr. = 27·200 parts, or 1 part = 0·01029 gr.							

Between the 16th of July and the 30th of that month there is a fortnight in which the balance has not been employed. As this interval may have affected the sensibility of the balance, it seems preferable not to take a mean of all the observations, but to divide them into two groups; one from June 28 to July 16, the other from July 30 to August 6. Indeed it appears, by inspection, that to 0·02 gr. in the last period belong more parts than in the former. We shall consequently employ the two values

from June 28 to July 16, 1 part = 0·01169 gr.
from July 30 to Aug. 6, 1 part = 0·01029 gr. } ROBINSON's balance.

10. The barometer used in the course of these comparisons was lent by Messrs. TROUGHTON and SIMMS to Captain NEHUS, because the instrument ordered before for that purpose was not finished when Captain NEHUS arrived in England. He received it but some days before his return. Captain NEHUS compared the instrument of Messrs. TROUGHTON and SIMMS 19 times, from June 17 to June 27, with the barometer of the Royal Society, and found that 0·066 inch must be added to its indications in order to correspond with those of the instrument of the Royal Society.

This instrument, after the experiments were completed, was returned to the owners, and could of course not be immediately compared with my standard barometer of BUZENGEIGER, whose tube has an interior diameter of nearly 8 French lines. It was, however, compared with my standard by means of a mountain barometer of DOLLOND, which Captain NEHUS brought over, and which had in London been compared with the instrument of Messrs. TROUGHTON and SIMMS. According to these comparisons 0·057 inch must be added to its indications to reduce them to *absolute* heights. I have adopted this correction, which differs 0·009 inch from that given by the barometer of the Royal Society, because my standard is furnished with an apparatus serving to verify the position of the microscope with regard to the divided scale, and because in tubes of so large a diameter there can be no uncertainty in the value of the small correction for capillarity. For the comparison of the two pounds, it is of little consequence what correction is employed, + 0·066 inch or + 0·057 inch. It is unnecessary to add, that +0·057 inch involves the correction for capillarity and for the zero point of the scale; so that after having applied it, the heights given by the barometer may be considered as *absolute* heights.

11. The thermometers employed were small thermometers with ivory scales made by Messrs. TROUGHTON and SIMMS, both suspended in the case of the balance near to the two ends of the beam. The thermometer marked L was at the left hand of the observer, the thermometer marked R at the right hand. I could determine only the corrections of the thermometer R, because the thermometer L was found to be broken when it arrived in Altona. Of course all its indications ought to be left out; and the temperature, during the observations, is only to be taken from the readings of the thermometer R.

For ascertaining its corrections, I employed two excellent standard thermometers; one the present of the late Mr. TROUGHTON, which he had constructed expressly for me, the other the present of Professor BESSEL, the corrections of which he had with great care determined, according to his method. Both standards have the FAHRENHEIT scale. The scale of the thermometer R being of ivory, I could not compare it with the standards in water, but I brought its bulb in contact with the bulb of the standards, and enveloped both bulbs in a thick cover of down. I found by

TROUGHTON's standard	at 69°·8	the corr. of R	= − 0°·70,	by 16 comp. made on 4 days.
	at 63·3	———	= − 0·61,	by 16 comp. made on 7 days.
BESSEL's standard	at 64·2	———	= − 0·64,	by 10 comp. made on 5 days.

The correction given by BESSEL's standard agrees within 0°·02 with that given by TROUGHTON's standard. Upon these data I have constructed the following small Table of corrections to be applied to the thermometer R.

	Correction.		Correction.
63°	= − 0°·61	67°	= − 0°·66
64	= − 0·62	68	= − 0·68
65	= − 0·63	69	= − 0·69
66	= − 0·65	70	= − 0·70

12. It remains only to state how the observed heights of the barometer have been reduced to absolute heights at the temperature of 32°. I have for this purpose employed a table, which will be given in my "Jahrbuch für 1837," constructed upon the formula,

$$\text{Barometer reduction to } 32^{\circ} = h\,\frac{m\,(t - 32^{\circ}) - s\,(t - 62^{\circ})}{1 + m\,(t - 32^{\circ})},$$

where h denotes the height read off in English inches from the brass scale, which represents English inches at the temperature of 62°, m the expansion of the mercury in volume for 1° of Fahrenheit (= 0·0001001), and s the expansion of the brass scale in length for 1° of Fahrenheit (= 0·000010434). The temperature of the brass scale and of the mercury is supposed to be the same.

13. After having stated all the elements necessary for the reduction of the observations before given, I may now give them reduced in a general synopsis.

	No. of Comp.	Comparisons.	b.	t.		No. of Comp.	Comparisons.	b.	t.
With Ramsden's Balance.									
1829.		gr.		°	1829.		gr.		°
June 21.	8	S.^{p} = U − 0·00811	29·683	67·07	June 29.	10	S.^{p} = U − 0·00927	29·630	66·34
23.	16	S.^{p} = U − 0·00764	29·832	66·90	July 1.	10	S.^{p} = U − 0·01459	29·498	65·84
24.	16	S.^{p} = U − 0·00637	29·922	67·22	4.	10	S.^{p} = U − 0·00930	29·507	65·01
24.	10	S.^{p} = U − 0·00652	29·964	67·32	5.	10	S.^{p} = U − 0·01229	29·495	64·12
28.	10	S.^{p} = U − 0·01311	29·328	68·51					
Mean of 100		S.^{p} = U − 0·00940 gr.	29·677	66°·54					
With Robinson's Balance.									
June 28.	10	S.^{p} = U − 0·01882	29·413	69·10	July 9.	30	S.^{p} = U − 0·00918	29·681	65·12
29.	10	S.^{p} = U − 0·00882	29·607	66·14	Aug. 1.	20	S.^{p} = U + 0·00044	30·141	64·07
July 1.	10	S.^{p} = U − 0·02180	29·392	66·04	2.	20	S.^{p} = U − 0·00281	30·146	65·35
4.	20	S.^{p} = U − 0·01458	29·481	64·67	3.	30	S.^{p} = U − 0·00473	29·910	65·09
5.	20	S.^{p} = U − 0·01119	29·560	64·57	4.	30	S.^{p} = U − 0·00518	29·686	64·64
Mean of 200		S.^{p} = U − 0·00815 gr.	29·745	65°·16					

The means are taken according to the number of the comparisons of each day *: b is the absolute height of the barometer reduced to 32°, and t is the temperature of the air and weights as given by the Thermometer R; its indications being corrected by the Table before given (11.).

The general mean of all the 300 comparisons made (those made by Robinson's balance having double weight because they are double in number of those made by Ramsden's balance) is

$$S.^p = U - 0{\cdot}00857 \text{ gr.} \qquad b = 29{\cdot}722, \quad t = 65^\circ{\cdot}62.$$

14. At the same time that Captain Nehus compared my platina copy S.^{p} with the imperial standard, he compared also the first brass copy (K.) sent by Captain Kater (and compared, as before stated, several times by him with his two copies,) with the imperial standard. I begin by giving these observations as they were made, and shall afterwards reduce them in the same manner as the comparisons of the platina pound.

The same balances, the same barometer, and the same thermometers were used; as indeed they were at all the weighings made in Somerset House.

* If the mean result of several days be R, R′, R″, ... , and the corresponding numbers of comparisons made on each day n, n', n'', ... , the mean of all the results is $= \dfrac{R\,n + R'\,n' + R''\,n'' + \ldots\ldots}{n + n' + n'' + \ldots\ldots}$.

With RAMSDEN's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
1.	K = U + 5·19 pt.	29·748	68°·7	66°·8	67°·0	5.	K = U + 5·08 pt.	......	......	66°·9	67°·1
2.	K = U + 1·68					6.	K = U + 4·03				
3.	K = U + 4·87					7.	K = U + 4·00				
4.	K = U + 3·40					8.	K = U + 4·33	......	......	67·0	67·4

1829, June 21. Mean of 8 ...... K = U + 4·0725 pt. Barom. 29·748 Att. Th. 68°·7 Th. L. 66°·9 Th. R. 67°·17

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
9.	K = U + 6·075	29·753	66·4	65·4	65·7	14.	K = U + 4·375				
10.	K = U + 4·125					15.	K = U + 5·600	......	......	66·0	66·4
11.	K = U + 4·825	......	......	65·8	66·0	16.	K = U + 4·000				
12.	K = U + 4·800					17.	K = U + 2·550	......	......	66·0	66·3
13.	K = U + 4·700	......	......	66·0	66·0	18.	K = U + 3·350	29·736	67·3		

1829, June 22. Mean of 10 ...... K = U + 4·440 pt. Barom. 29·745 Th. Att. 66°·9 Th. L. 65°·84 Th. R. 66°·08

With ROBINSON's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
1.	K=U+0·03 gr.−0·40 pt.	29·812	63·6	64·2	64·5	17.	K=U+0·03 gr.+0·05 pt.				
2.	K=U+0·03−0·70					18.	K=U+0·03+0·10				
3.	K=U+0·03−0·05					19.	K=U+0·03−0·25				
4.	K=U+0·03−0·60					20.	K=U+0·03+0·25				
5.	K=U+0·03+0·40					21.	K=U+0·03+0·05				
6.	K=U+0·03+0·45					22.	K=U+0·03+0·65				
7.	K=U+0·03+1·20	29·713	63·9	64·6	64·6	23.	K=U+0·03+0·10				
8.	K=U+0·03+0·25					24.	K=U+0·03+1·45				
9.	K=U+0·03−0·10					25.	K=U+0·03+0·55	......	......	65·5	65·7
10.	K=U+0·03+0·05					26.	K=U+0·03+0·30				
11.	K=U+0·03+0·35					27.	K=U+0·03+0·10				
12.	K=U+0·03+0·20					28.	K=U+0·03+0·70				
13.	K=U+0·03+0·35					29.	K=U+0·03+0·05				
14.	K=U+0·03+0·75					30.	K=U+0·03+0·45				
15.	K=U+0·03+0·40	......	......	64·8	64·9	31.	K=U+0·03+0·20				
16.	K=U+0·03+0·55					32.	K=U+0·03+0·95	29·634	64·5	65·4	65·7

1829, July 7. Mean of 32 ..... K = U + 0·03 gr. + 0·2750 pt. Barom. 29·720 Th. Att. 64°·0 Th. L. 64°·9 Th. R. 65°·08

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
33.	K=U+0·03+1·65	29·800	65·4	66·0	66·2	40.	K=U+0·03+0·60				
34.	K=U+0·03+1·45					41.	K=U+0·03−0·15				
35.	K=U+0·03+1·00					42.	K=U+0·03−0·80				
36.	K=U+0·03+0·60					43.	K=U+0·03−0·55				
37.	K=U+0·03+0·70					44.	K=U+0·03 0·00				
38.	K=U+0·03−0·45					45.	K=U+0·03 0·00				
39.	K=U+0·03 0·00					46.	K=U+0·03+0·25	29·828	65·0	66·2	66·4

1829, July 9. Mean of 14 ...... K = U + 0·03 gr. + 0·3071 pt. Barom. 29·814 Att. Th. 65°·2 Th. L. 66°·1 Th. R. 66°·3

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
47.	K=U+0·03+1·10	29·481	64·2	65·0	65·2	55.	K=U+0·03−0·15	......	......	65·4	65·7
48.	K=U+0·03+0·20					56.	K=U+0·03+0·15				
49.	K=U+0·03−0·05					57.	K=U+0·03+0·70				
50.	K=U+0·03+0·50					58.	K=U+0·03+0·40				
51.	K=U+0·03+0·30					59.	K=U+0·03+1·00				
52.	K=U+0·03+0·70					60.	K=U+0·03−0·05				
53.	K=U+0·03+0·60					61.	K=U+0·03+0·60				
54.	K=U+0·03+0·45					62.	K=U+0·03+0·15	29·414	64·8	65·7	65·9

1829, July 11. Mean of 16 ...... K = U + 0·03 gr. + 0·4125 pt. Barom. 29·447 Att. Th. 64°·5 Th. L. 65°·37 Th. R. 65°6

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
63.	K=U+0·03+0·80	29·672	64·8	65·0	65·3	69.	K=U+0·03+0·10	......	......	66·0	66·2
64.	K=U+0·03+0·40					70.	K=U+0·03+0·25				
65.	K=U+0·03+0·25					71.	K=U+0·03+0·35				
66.	K=U+0·03+0·15					72.	K=U+0·03−0·20				
67.	K=U+0·03+0·55					73.	K=U+0·03+1·30				
68.	K=U+0·03+0·50					74.	K=U+0·03−0·05				

1829, July 30. Mean of 12 ...... K = U + 0·03 gr. + 0·3667 pt. Barom. 29·672 Att. Th. 64·8 Th. L. 65·5 Th. R. 65·75

15. These weighings give, using the same reductions as before for P, and taking the mean according to the number of comparisons:

	No. of Comp.	Comparisons.	b.	t.		No. of Comp.	Comparisons.	b.	t.
1829. June 21.	8	K = U + 0·03292 gr.	29·698	66°·51	1829. July 9.	14	K = U + 0·03359 gr.	29·773	65°·64
22.	10	K = U + 0·03589	29·700	65·43	11.	16	K = U + 0·03482	29·409	64·96
July 7.	32	K = U + 0·03321	29·683	64·45	30.	12	K = U + 0·03378	29·633	65·10
Mean of 92 ...... K = U + 0·03389 gr. b. 29·646 t. 65°·09									

This result differs considerably from that given by Capt. Kater by his last comparisons with his two weights, by which he found K = U + 0·0259 gr. (see § 5.). It is already there stated that K seems not to have undergone any change in the mean time; and its invariability is yet further proved by fifty comparisons made in October 1829 and February 1830 with K^n, by which I found

	No. of Comp.	Comparisons.		No. of Comp.	Comparisons.
1829. Oct. 7.	20	K = K^n + 0·02079	1830. Feb. 13.	12	K = K^n + 0·01916
1830. Feb. 11.	14	K = K^n + 0·01921	14.	4	K = K^n + 0·02106
Mean of 50 ...... K = K^n + 0·01998 gr.					

This differs but 0·0002 grains, and of course nothing of importance, from the result obtained in 1828, when I found (see § 3.) K = K^n + 0·0198 grain.

16. After having compared K with the imperial standard troy pound, Capt. Nehus compared with it a brass pound which I had ordered at Mr. Robinson's. I denote this pound by S.^{b}. The comparisons were made with the same instruments as before specified, which served for all the weighings made in London. They are as follow.

With Robinson's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	gr. pt.						gr. pt.				
1.	S.^{b}=U−0·01+0·25	30·225	64°·2	65°·0	65°·2	11.	S.^{b}=U−0·01−0·30		°	°	°
2.	S.^{b}=U−0·01 0·00					12.	S.^{b}=U−0·01−0·10				
3.	S.^{b}=U−0·01 0·00					13.	S.^{b}=U−0·01+0·15	......	......	65·9	66·0
4.	S.^{b}=U−0·01−0·35					14.	S.^{b}=U−0·01−0·15				
5.	S.^{b}=U−0·01−0·30					15.	S.^{b}=U−0·01+0·15				
6.	S.^{b}=U−0·01−0·15	.....	......	65·8	66·0	16.	S.^{b}=U−0·01−0·10				
7.	S.^{b}=U−0·01+0·20					17.	S.^{b}=U−0·01+0·60				
8.	S.^{b}=U−0·01−0·30					18.	S.^{b}=U−0·01+0·10				
9.	S.^{b}=U−0·01−0·20					19.	S.^{b}=U−0·01+0·25				
10.	S.^{b}=U−0·01−0·25					20.	S.^{b}=U−0·01−0·90	30·188	64·6		
1829, August 2.	Mean of 20 ...... S.^{b} = U − 0·01 gr. − 0·070 pt.	Barom. 30·2065	Att. Th. 64°·4	Th. L. 65°·57	Th. R. 65°·73						
21.	S.^{b}=U−0·01+0·30	29·585	63·0	64·2	64·5	31.	S.^{b}=U−0·01+0·65				
22.	S.^{b}=U−0·01−0·85					32.	S.^{b}=U−0·01+0·15				
23.	S.^{b}=U−0·01−0·25					33.	S.^{b}=U−0·01+0·40				
24.	S.^{b}=U−0·01−0·35					34.	S.^{b}=U−0·01−0·80	......	......	64·8	65·0
25.	S.^{b}=U−0·01 0·00					35.	S.^{b}=U−0·01+0·25				
26.	S.^{b}=U−0·01+0·15	......	......	64·8	65·0	36.	S.^{b}=U−0·01+0·10				
27.	S.^{b}=U−0·01+0·10					37.	S.^{b}=U−0·01+0·75				
28.	S.^{b}=U−0·01 0·00					38.	S.^{b}=U−0·01+0·55				
29.	S.^{b}=U−0·01+0·30					39.	S.^{b}=U−0·01−0·65				
30.	S.^{b}=U−0·01−0·50					40.	S.^{b}=U−0·01−0·10	29·940	63·8		
1829, August 5.	Mean of 20 ...... S.^{b} = U − 0·01 gr. + 0·010 pt.	Barom. 29·7625	Att. Th. 63°·4	Th. L. 64°·6	Th. R. 64°·83						

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	gr. pt.		°	°	°		gr. at.		°	°	°
41.	S.^{b}=U−0·01+0·30	30·036	63·2	64·2	64·7	51.	S.^{b}=U−0·01−0·10				
42.	S.^{b}=U−0·01+0·25					52.	S.^{b}=U−0·01+0·40				
43.	S.^{b}=U−0·01+0·40					53.	S.^{b}=U−0·01+0·35				
44.	S.^{b}=U−0·01 0·00					54.	S.^{b}=U−0·01−0·45				
45.	S.^{b}=U−0·01−0·55	......	......	64·8	65·0	55.	S.^{b}=U−0·01−0·25				
46.	S.^{b}=U−0·01−0·45					56.	S.^{b}=U−0·01 0·00				
47.	S.^{b}=U−0·01−0·05					57.	S.^{b}=U−0·01−0·00				
48.	S.^{b}=U−0·01+0·30					58.	S.^{b}=U−0·01−0·20				
49.	S.^{b}=U−0·01−0·25					59.	S.^{b}=U−0·01+0·45				
50.	S.^{b}=U−0·01−0·95					60.	S.^{b}=U−0·01 0·00				

1829, August 6. Mean of 20 ...... S.^{b} = U − 0·01 gr. − 0·040 pt. Barom. 30·036 Att. Th. 63°·2 Th. L. 64°·5 Th. R. 64°·85

These weighings give (using the same elements of reduction as before, and giving to *b* and *t* the significations already mentioned, as well as taking the means according to the number of observations), the following results:

	No. of Comp.	Comparisons.	*b.*	*t.*
1829.		gr.		°
August 2.	20	S.^{b} = U − 0·01072	30·167	65·09
5.	20	S.^{b} = U − 0·00990	29·727	64·20
6.	20	S.^{b} = U − 0·01041	30·000	64·22
Mean of 60		S.^{b} = U − 0·01034	29·965	64·50

17. These were the comparisons of my own pounds, which Captain Nehus made at Somerset House. He compared also a platina troy pound belonging to the Royal Society, and the brass pound of the Royal Mint, with the now lost imperial standard troy pound. The platina troy pound was made, upon the President's orders, by Mr. Cary *. I shall first give its comparisons with the imperial standard troy pound, premising only that I designate it by the letters RS.

With Robinson's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	gr. pt.		°	°	°		gr. pt.		°	°	°
1.	RS=U−0·02+1·10	29·380	64·8	65·6	65·9	11.	RS=U−0·02+0·80				
2.	RS=U−0·02+1·10					12.	RS=U−0·02+0·85				
3.	RS=U−0·02+0·05					13.	RS=U−0·02+0·65				
4.	RS=U−0·02+1·35					14.	RS=U−0·02+0·50				
5.	RS=U−0·02+0·55					15.	RS=U−0·02+0·70	......	......	66·3	66·6
6.	RS=U−0·02+0·55					16.	RS=U−0·02+1·20				
7.	RS=U−0·02+1·55	......	......	66·0	66·2	17.	RS=U−0·02+0·60				
8.	RS=U−0·02+0·95					18.	RS=U−0·02+1·85				
9.	RS=U−0·02+0·30					19.	RS=U−0·02+0·70				
10.	RS=U−0·02+0·30					20.	RS=U−0·02+0·95	29·434	65·2	66·0	66·5
	1829, July 12. Mean of 20 ...... RS = U − 0·02 gr. + 0·825 pt.	Barom. 29·407	Att. Th. 65°·0	Th. L. 66°·0	Th. R. 66°·3						
	pt.										
21.	RS = U − 0·00	29·810	66·0	67·0	67·2	28.	RS = U − 0·10				
22.	RS = U − 0·60					29.	RS = U 0·00				
23.	RS = U − 0·05					30.	RS = U − 0·50				
24.	RS = U 0·00					31.	RS = U − 0·35				
25.	RS = U + 0·10					32.	RS = U 0·00				
26.	RS = U − 0·35	......	......	67·5	67·8	33.	RS = U + 0·30				
27.	RS = U + 0·15					34.	RS = U − 0·30	29·832	66·8	67·8	68·1
	1829, July 14. Mean of 14 ...... RS = U − 0·12143 pt.	Barom. 29·821	Att. Th. 66°·4	Th. L. 67°·4	Th. R. 67°·7						

* For the purpose of making this new platina pound, Mr. Cary was furnished with some of the platina which Dr. Wollaston had given to the Royal Society: but it seems that Mr. Cary did not employ *this* platina, but used some other kind; the reason for which has not been sufficiently explained.—F. Baily.

With Robinson's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	gr. pt.		°	°	°		pt.		°	°	°
35.	RS=U−0·02+1·25	29·932	66·8	67·5	67·8	43.	RS=U −0·20				
36.	RS=U−0·02+1·15					44.	RS=U −0·60				
37.	RS=U +0·05					45.	RS=U −0·05				
38.	RS=U −0·25					46.	RS=U −0·15				
39.	RS=U −0·25					47.	RS=U −0·05				
40.	RS=U −0·50	......	......	67·9	68·1	48.	RS=U −0·50	......	......	68·2	68·7
41.	RS=U −0·25					49.	RS=U −0·05				
42.	RS=U −0·40					50.	RS=U −0·60	29·902	67·0	68·1	68·6

1829, July 15. Mean of 16 ...... RS = U − 0·0025 gr. − 0·0875 pt. Barom. 29·917 Att. Th. 66°·9 Th. L. 67°·92 Th. R. 68°·3

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
51.	RS = U + 0·30	29·884	66·3	67·2	67·7	61.	RS = U − 0·20				
52.	RS = U − 0·40					62.	RS = U − 0·45				
53.	RS = U − 0·35					63.	RS = U 0·00				
54.	RS = U − 0·40					64.	RS = U − 0·25	......	......	67·9	68·1
55.	RS = U − 0·40					65.	RS = U − 0·20				
56.	RS = U − 1·10	......	......	67·6	68·0	66.	RS = U − 0·50				
57.	RS = U − 1·00					67.	RS = U − 0·30				
58.	RS = U − 0·60					68.	RS = U − 0·05				
59.	RS = U − 1·10					69.	RS = U − 0·50				
60.	RS = U − 0·10					70.	RS = U − 0·55	29·884	67·0	67·9	68·2

1829, July 16. Mean of 20 ...... RS = U − 0·4075 pt. Barom. 25·884 Att. Th. 66°·65 Th. L. 67°·65 Th. R. 68°·0

With Ramsden's Balance.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
71.	RS = U + 1·30	30·224	63·2	64·8	65·0	86.	RS = U − 0·55				
72.	RS = U + 0·70					87.	RS = U + 0·15				
73.	RS = U + 0·80					88.	RS = U + 0·10				
74.	RS = U + 0·60					89.	RS = U 0·00				
75.	RS = U + 0·35					90.	RS = U + 0·25				
76.	RS = U + 0·10	......	......	65·0	65·1	91.	RS = U + 1·60				
77.	RS = U + 1·94					92.	RS = U + 0·20	......	......	65·4	65·6
78.	RS = U + 0·45					93.	RS = U + 0·80				
79.	RS = U − 0·15					94.	RS = U + 0·10				
80.	RS = U + 0·20					95.	RS = U + 0·45				
81.	RS = U + 0·65					96.	RS = U − 0·05				
82.	RS = U + 0·10					97.	RS = U 0·00				
83.	RS = U + 0·20					98.	RS = U + 0·05				
84.	RS = U + 0·10	......	......	65·1	65·2	99.	RS = U + 0·50				
85.	RS = U + 0·65					100.	RS = U − 0·05	30·225	64·2		

1829, August 2. Mean of 30 ...... RS = U + 0·381667 pt. Barom. 30·2245 Att. Th. 63°·7 Th. L. 65°·07 Th. R. 65°·22

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
101.	RS = U − 0·30	29·764	63·6	65·0	65·0	121.	RS = U + 0·60				
102.	RS = U − 1·50					122.	RS = U − 0·80				
103.	RS = U + 0·45					123.	RS = U 0·00				
104.	RS = U − 0·05					124.	RS = U + 0·45				
105.	RS = U 0·00					125.	RS = U − 0·50				
106.	RS = U + 1·25					126.	RS = U − 0·10				
107.	RS = U + 0·20					127.	RS = U − 0·70				
108.	RS = U + 0·15					128.	RS = U + 0·05	......	......	65·2	65·4
109.	RS = U − 0·20					129.	RS = U 0·00				
110.	RS = U − 0·05	......	......	65·0	65·2	130.	RS = U − 0·55				
111.	RS = U − 0·35					131.	RS = U − 0·10				
112.	RS = U − 0·50					132.	RS = U + 0·30				
113.	RS = U − 0·55					133.	RS = U − 0·35				
114.	RS = U − 0·05					134.	RS = U − 0·40				
115.	RS = U + 0·15					135.	RS = U − 0·10				
116.	RS = U − 1·00					136.	RS = U + 1·40	......	......	65·4	65·5
117.	RS = U + 0·35					137.	RS = U + 0·20				
118.	RS = U − 0·10	......	......	65·0	65·1	138.	RS = U − 1·35				
119.	RS = U + 1·00					139.	RS = U 0·00				
120.	RS = U + 0·50	29·742	64·0			140.	RS = U − 0·90	29·732	64·0		

1829, August 4. Mean of 40 ...... RS = U − 0·08625 pt. Barom. 29·746 Att. Th. 63°·87 Th. L. 65°·12 Th. R. 65°·24

These weighings give (using the same elements of reduction as before, giving to b and t the signification already explained, and taking the means according to the number of observations,) as follow:

1829.	No. of Comp.	Comparisons.	*b.*	*t.*	1829.	No. of Comp.	Comparisons.	*b.*	*t.*
		gr.		°			gr.		°
July 2.	20	RS = U − 0·01036	29·368	65·65	July 16.	20	RS = U − 0·00476	29·839	67·32
14.	14	RS = U − 0·00142	29·777	67·03	Aug. 2.	30	RS = U + 0·00437	30·186	64·59
15.	16	RS = U − 0·00352	29·871	67·62	4.	40	RS = U − 0·00099	29·709	64·61
		Mean of 140 ...... RS = U − 0·00205 gr.	29·806 *b.*	65°·73 *t.*					

The first four days ROBINSON'S balance is used; the last two, RAMSDEN'S balance.

18. The last pound which Captain NEHUS compared with the now lost imperial standard troy pound, was that of the Royal Mint. It is marked "Ty Pd 1824", and the same that Captain KATER designates by No. 3.* Mr. (afterwards Sir JOHN) BARTON, of the Royal Mint, had the kindness to bring it to Somerset House, and it was there compared in his presence. The pound appeared in high preservation, and bore no marks of any oxydation whatever. It is needless to repeat that the same barometer and thermometers were used. The balance was that of ROBINSON. I designate this pound by the mark RM, since it belongs to the Royal Mint.

No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.	No.	Comparisons.	Barom.	Attached Therm.	Therm. L.	Therm. R.
	pt.			°	°		pt. gr.		°	°	°
1.	RM = U + 0·20	......	......	65·8	66·1	9.	RM=U+0·90				
2.	RM = U + 0·65					10.	RM=U+1·20	....	....	66·5	66·8
3.	RM = U + 0·75					11.	RM=U 0·00+0·01				
4.	RM = U + 0·60					12.	RM=U−0·30+0·01				
5.	RM = U + 1·10					13.	RM=U+0·45+0·01				
6.	RM = U + 0·55					14.	RM=U+0·20+0·01				
7.	RM = U + 1·30					15.	RM=U−0·05+0·01				
8.	RM = U + 0·20					16.	RM=U+0·20+0·01	29·776	65·0	66·2	66·8
	1829, July 30.		Mean of 16 ...... RM = U + 0·496875 pt. + 0·00375 gr.					29·776 Barom.	65°·0 Att. Th.	65°·17 Th. L.	66°·57 Th. R.

This gives, reduced for the mean of 16 comparisons,

RM = U + 0·00887 gr. 29·679 *b.* 65°·91 *t.*

Captain KATER found in 1824 †, RM = U + 0·0021 gr. The difference is 0·0068 gr.

19. I shall now put all the results obtained by Captain NEHUS in London in one view.

No. of Comp.	Comparisons.	*b.*	*t.*
	gr.		°
92	K = U + 0·03389	29·646	65·09
300	S.p = U − 0·00857	29·722	65·62
60	S.b = U − 0·01034	29·965	64·50
140	RS = U − 0·00205	29·806	65·73
16	RM = U + 0·00887	29·679	65·91

* Philosophical Transactions, 1826, p. 12. 17, 18.

† Ibid., p. 18.

20. It would appear, from an inspection of these results, that the true weight of the now destroyed imperial standard troy pound might, even after its loss, be very well ascertained. There are five several pounds which were compared with it only six years ago with extraordinary care; these five pounds are still extant and in good preservation; the number of the comparisons exceeds 600, and these comparisons are made with excellent balances, and by a skilful and careful observer, who devoted, during several months, his whole time and attention to them. There remains, therefore, only (since each body weighed in air loses as much of its weight as the volume of air weighs, which is displaced by the body,) to add on both sides of the above 5 equations the weights of these displaced volumes of air: on one side the weight of the volume of air displaced by the copy, on the other side the weight of the volume of air displaced by the imperial standard troy pound; or to reduce (as it is generally called) these weighings to a vacuum. Indeed, if it had been possible to weigh the bodies in a vacuum, their weights would have sustained no losses, because there was nothing which they could displace, and the difference of weight indicated by the balance would have been their *true* difference of weight. This true difference of weight is evidently likewise obtained, when, by addition of the before-mentioned weights of the displaced volumes of air, the weighings made in air are corrected for the losses which the weights of the bodies necessarily suffered from the bodies being obliged to put aside the medium in which they were weighed: so that both modes of proceeding lead to the same result, viz. to the true difference of weight of the bodies.

Unhappily one of the most essential elements to calculate the volume of the now lost imperial standard troy pound, and of course to calculate the weight of an equal volume of air displaced by it, is still unknown; I mean *the specific gravity* of that pound. Even the metal of which it was made is uncertain. It is declared to be a *brass* pound by the Act of Parliament 5 George IV. chap. lxxiv. §. 4, as follows:

"And be it further enacted, That from and after the first day of May 1825, the "standard *brass* weight of one pound troy weight, made in the year 1758, now in the "custody of the Clerk of the House of Commons, shall be, and the same is hereby "declared to be, the original and genuine standard measure of weight: and that "such *brass* weight shall be, and the same is hereby declared to be, the unit or only "standard measure of weight, from which all other weights shall be derived, com-"puted and ascertained."

But as a law never pretends, nor can pretend, to decide upon the physical qualities of bodies, the expression *brass weight* must be understood as suggested to the legislators by those to whom the adjusting of the weights was committed, and states of course only *their* private opinion about the metal of which the pound was made; or, as this department fell under Captain Kater's care, we may consider the words *brass pound, brass weight,* as the expression of *his* private opinion. This is the more likely, because he seems to have considered it not only as brass, but as brass of the *same* specific gravity with the metal of the new pounds compared by him with the imperial standard. Indeed, he has nowhere noted (or, if he had noted, has nowhere published,) the state of the barometer and thermometer at the time of his comparisons; an omission which is only allowable when both weights are supposed to be of the same specific gravity and expansion *.

On the other side, Captain NEHUS, who had the imperial standard troy pound several months under his eyes, is decidedly of opinion that it was *not* of brass. He declares that its dark brown colour left no doubt that it must be of copper or bell-metal. Mr. ROBINSON, whose decision was called in aid, thought it (from the form of the rands, which the impression of the stamps had raised, and from the fine porosity of the surface,) to be of copper.

21. We have thus private opinion against private opinion, and the question still remains undecided. Under the impression that in the Parliamentary Reports about the year 1758, when the imperial standard troy pound was made, something more decisive respecting the metal of that pound might be found, I carefully perused the second volume of the "*Reports from Committees of the House of Commons.* Re-"printed (1803) by order of the House. Miscellaneous Subjects 1738—1765.—Folio." It contains two Reports from the Committees appointed to inquire into the original standards of weights and measures, both presented by Lord CARYSFORT: the first on May 26, 1758, the second on April 11, 1759. It appears that Mr. HARRIS, then Assay-Master of the Mint, presented to the first Committee three troy pounds made under his direction, whose weight was determined by a mean taken from the best old standards existing at that time, as described in the following words (p. 437 b): viz. "Therefore to ascertain the troy pound, according to the aforesaid experiments," (the mean taken from several standards,) "your Committee directed three several "troy pounds to be made under the direction of Mr. HARRIS, to be marked as "follows:"

[Here a rough sketch is inserted, showing the form of the imperial standard troy pound, and the marks stamped upon it.]

"and these have been accordingly made and adjusted with very curious and exact "scales of his at the Mint, and found to agree with the result of the experiments "made by the Committee: one of these weights is also produced herewith."

* Supposing BATE's metal, from which the new pounds were made, to be 8·0 specific gravity, and the imperial standard to be 8·1 (being a difference of only 0·1 in specific gravity from BATE's pounds), this would have required under common atmospheric circumstances (barom. 30·0, therm. 62°) a correction of 0·0109 grains to be applied to the comparisons, in order to obtain the true difference of weight. It is evident that Captain KATER, who gave his comparisons in *ten thousandth* parts of a grain, would not have neglected corrections which for so small a difference in the specific gravity affect the results even to *hundredths* of a grain, if he had not considered the metal of the old pound identical with that of the new pounds. But of this he could know nothing for certain.

This pound produced to the Committee was the now lost imperial standard troy pound, as appears from the 8th Resolution of the Committee (p. 439 a.), which proposes: "That the standard of weight ought to be the pound herewith delivered, "described in this Report, and made upon the examination and review of the several "present standard troy weights therein mentioned; and that the 12th part of the said "pound should be an ounce, the 20th part of such ounce a pennyweight, and the 24th "part of such pennyweight a grain."

This 8th Resolution of the Committee was agreed to by the House on the 2nd of June, 1758 (p. 463 a.). The pound itself was presented to the House (p. 456 b.*) in the preceding Session (April 11, 1759), and probably remained under the custody of the Clerk of the House of Commons until it was destroyed by the late fire. In all these statements there is not the least mention made of the metal of which it consisted, nor have I been able to find anything *decisive* about this point elsewhere in either of the Reports.

22. It appears, indeed, (p. 428 a.) that the standard weights of Guildhall were of *brass*; that the Charter of the Founders' Company, September 18, 1614, 12 James I., speaks of *brass* weights (p. 428 b.); that the weights in the Court of Receipt of the Exchequer were of *brass* (p. 447 b.). So that there is a great degree of probability that Mr. HARRIS, who adjusted his new weights upon those mentioned, made them also of brass: but this probability is somewhat lessened by the Report of the second Committee. This Committee took into its special consideration (p. 459 b.) the material of which the weights for the future were to be made; and it seems natural they should have decided for brass, if the new standard, already declared such by Parliament, (June 2nd, 1758,) or, what is the same, the now lost imperial standard troy pound, had been of brass: but they resolve only (p. 461 b.): "That models or pat- "terns of measures of length and weights established as the genuine standards of "the kingdom should be made of *fine hard metal*, and deposited in the same place "and kept in the same manner as the trial-pieces for the coin, used on trials of the "Pix, are preserved in the Exchequer."

What is understood by *hard metal* appears (p. 460 a.) in the same Report, where the Committee decides, that "the smaller weights, from a pound through all its parts, "are most conveniently made of *copper*, *brass*, or other such hard metal;" where *copper* is even put in the first place as significative of the term *hard metal*, though, after the alphabetic order, *brass* ought to have preceded.

23. There is however, in the Report of this second Committee, (p. 457 a.) a statement which seems at first view to lead us to the knowledge at least of the metal, if not of the specific gravity of the lost standard. It is as follows: "Mr. HARRIS has "procured, by order of your Committee, two sets of the following multiples of the

* It seems that the pound was delivered to the House the 2nd of June 1758. "The Resolution of the former "Committee, agreed to by the House 2nd June 1758, ascertains the standard of weight to be the pound *then* "delivered to the House," &c. (p. 456 b.)

"standard pound, all of *fine brass*, which he will adjust with an apparatus, also "contrived on purpose, as soon as possible, viz.

" 2 pounds,
" 4 ditto,
" 8 ditto,
" 16 ditto,
" 32 ditto."

And we might consider this statement nearly as a proof that the imperial standard was also of the same *fine brass*, if the multiples might be supposed to be made *at the same time* with the standard; because it is highly probable that Mr. HARRIS, under whose direction the three single pounds and their multiples were made, would under these circumstances have made them all of *the same* metal. But it appears from the Report of the former Committee that the multiples *did not exist* when the three weights of one pound each were delivered to that Committee. For it states: "Your Committee intended to have hâd the parts and multiples of this troy pound "also made, but found that there were not instruments sufficiently exact for that "purpose; and the contriving and making of such would take up more time than "the probable continuance of this Session would permit: they have therefore *left it "for the future consideration of Parliament, whether anything of that nature should be "performed*." (p. 437 b.)

The multiples were of course not even begun to be made when the pounds were already finished; and though it seems probable that Mr. HARRIS would as nearly as possible have made them of the same kind of metal as that of which the three single pounds were made, yet this produces only *a probability* that the imperial standard was of brass; but nothing proves that Mr. HARRIS could procure afterwards *the same* fine brass of which the three single pounds were formerly made. Now it is generally known that brass, as a compound metal, varies in density according to the different proportions of the compounding metals adopted by the makers. (I have found differences in specific gravity in different kinds of brass from 7·9 to 8·4.) Of course, if it was even *proved*, as it is not, that the imperial standard was of brass, nothing would be gained by that proof, if we could not at the same time ascertain the identical brass from which it was made.

24. This we should be able to do with a degree of probability bordering almost on certainty, if the other two single pounds, of the three presented to the Committee, could be traced, and were still existing. Indeed it is in the highest degree probable that these three single pounds, made all under Mr. HARRIS's direction, adjusted and presented at the same time by him, were also of the same identical metal: it remains only to ascertain what became of them. The 7th Resolution of the second Committee (p. 463 a.) states: "That it is the opinion of this Committee, that the yard mentioned "in the 2nd Resolution of the former Committee upon the subject of weights and "measures, agreed to by the House the 2nd June 1758, being the standard of length,

"*and the pound mentioned in the* 8th Resolution of the former Committee upon the "subject of weights and measures, agreed to by the House the 2nd June 1758, being "the standard of weight, ought to be deposited in the Court of Receipt of the Exche- "quer, and there safely kept under the Seals of the Chancellor of the Exchequer, and "of the Chief Baron, and the Seal of Office of the Chamberlains of the Exchequer, "and not to be opened but by the order and in the presence of the Chancellor of the "Exchequer and the Chief Baron for the time being."

If this pound, intended to be deposited in the Exchequer according to the wishes of the Committee, might be supposed to be a different pound from that presented to the House, (p. 457 b.) and of course one of the two remaining of the three that were presented to the former Committee, we might know (provided the Resolution was agreed to by the House) where to look for a pound of the identical metal of the lost standard; but the obvious meaning of the words seems to be, that the pound presented to the House in the last Session (i. e. the imperial standard now lost) should not be kept there, because the Committee thought it safer in the custody of the Exchequer. This is the more probable, because the following 9th Resolution (p. 463 a.) says expressly that the standard yard mentioned in the 2nd Resolution of the former Committee, agreed to by the House June 2nd 1758, was at the moment when the last Committee formed their Resolutions "now" *in the custody of the Clerk of the House*, which is not said of the standard pound, of which mention is made immediately after the yard, and which certainly would have been added (since they are precise in their expressions) if the pound had been at that moment really in the custody of the said Clerk. The Committee had probably taken it back, in order to have the multiples adjusted, which was not yet done, (see the 9th Resolution, where the words "when the same [the multiples] *are* adjusted" imply that meaning: it is also expressly said, p. 457, that Mr. HARRIS *will* adjust the multiples *as soon as possible*;) and which could not be done without having the use of the standard; and proposed by their 7th Resolution not to restore it to the Clerk of the House, but to deposit it, after having used it, at the Exchequer, where they meant also to transfer the yard, which was still in the custody of the Clerk.

It is not stated that the House agreed to this Resolution: on the contrary, it appears that it did not, because both yard and pound were, when Captain KATER compared them, and before that time, in the custody of the Clerk of the House of Commons, where they were originally deposited.

25. The following Resolutions (the 8th and 9th) of the second Committee might also possibly be understood as referring to one of the two remaining pounds of Mr. HARRIS. The 8th Resolution (p. 463 a.) proposes as the most effectual means to ascertain uniformity in measures of length and weights to be used throughout the realm, "To appoint certain persons at one particular office, with clerks and workmen "under them, for the purpose only of sizing and adjusting, for the use of the sub- "jects, all measures of length, and all weights, being parts, multiples, or certain pro-

" portions of the standards, to be used for the future." To which the 9th Resolution (ibidem) adds: "That it is the opinion of this Committee, that a model or pattern of the " said standard yard mentioned in the 2nd Resolution of the former Committee, agreed " to by the House June 2, 1758, *and now in custody of the Clerk of the House,* and a model " or pattern of the standard pound, mentioned in the 8th Resolution of the former " Committee, agreed to by the House June 2nd, 1758, together with models or patterns " of the parts of the said pound now presented to the House, and also of the multiples " of the said pound mentioned in this Report (*when the same are adjusted*), should be " kept in the said office, in custody of the said persons to be appointed for sizing " weights and measures, under the seal of the Chief Baron of the Court of Exchequer " for the time being, to be opened only by order of the said Chief Baron in his pre- " sence, or the presence of one of the Barons of the Exchequer, on the application of " the said persons, for the purpose of correcting and adjusting, as occasion shall re- " quire, the patterns or models used at the said office for sizing measures of length " and weights delivered out to the subjects."

This model or pattern of the standard pound, to be kept in the proposed office, might be understood to design one of the remaining pounds of Mr. HARRIS; but if the Committee had had one of those pounds in view, they would probably have specified it as one of the three single pounds presented by Mr. HARRIS to the former Committee: moreover, the already alleged 1st Resolution of the second Committee (p. 461 b.) speaks of models or patterns of measures of length and weights as " to " be made." They were of course *not yet made,* and therefore cannot be supposed to design the two remaining pounds of Mr. HARRIS, which *were already made* and adjusted, and presented as such in the preceding year (1758) to the former Committee.

To repeat in few words the results of these inquiries, it appears,

1°. That the Reports of the Committees in 1758 and 1759 do not specify the metal of which the imperial standard troy pound was made.

2°. That there is a probability that it was made of brass, according to Captain KATER's opinion.

3°. That if it was even certain that it was of brass, we ought to know of what kind of brass it was made, as the specific gravity of that metal varies according to its composition.

4°. That it is in the highest degree probable that the two remaining pounds which Mr. HARRIS presented to the Committee in 1758, were of the identical metal of the standard, so that if we could discover if these pounds still exist, and where they exist, the requisite specific gravity of the lost imperial standard troy pound might be ascertained.

5°. That the Reports of the Committees in 1758 and 1759 contain nothing by which we can learn what has become of these two remaining pounds.

26. Although the Reports of the Committee which presented the imperial standard troy pound to the House contain nothing from which we can ascertain its specific

gravity, there appears from another quarter a new prospect to arrive at a knowledge of this most important point. The Report from the Select Committee of the House of Lords, appointed to consider the petition of the Directors of the Chamber of Commerce and Manufactures, established by royal charter in the City of Glasgow, taking notice of the Bill intituled "*An Act for ascertaining and establishing Uniformity of Weights and Measures*," and praying their Lordships to give the matter of their petition due consideration, and that they will introduce into the Bill such parts of the petition as shall to their Lordships appear likely to prove beneficial; together with *the Minutes of Evidence* taken before the said Committee, 1823, (Ordered by the House of Commons to be printed, March 2, 1824,) Folio, contains, p. 14, under the Minutes of Evidence, the examination of Dr. KELLY (May 31, 1823): where, upon the query, "What was effected with regard to the weights and measures by the Com- "mittee of 1758?" Dr. KELLY answers: "They ordered three several troy pounds "to be adjusted, under the direction of Mr. HARRIS, the then Assay-Master of the "Mint. One of these was placed in the custody of the Clerk of the House of Com- "mons; another was left with Mr. HARRIS, and is that now in the possession of "Mr. BINGLEY; and the third was, I understand, delivered to Mr. FREEMAN, weight- "maker to the Mint, the Exchequer, and the Bank of England, who used it as his "standard, and it is still so employed by his successor, Mr. VANDOME."

There is moreover on the same page 14 the following note:

"This weight [Mr. BINGLEY's pound] was produced to the Committee, at a sub- "sequent meeting, by Mr. BINGLEY, who said it had formerly belonged to Mr. HARRIS "when he held the situation of Assay-Master. There was a memorandum on the lid "of the box in which it was kept, stating that Mr. HARRIS had made from it the "pound weight which was placed in the custody of the Clerk of the House of Com- "mons by direction of the Committee of 1758, and which is called commonly the "Parliamentary pound."

If Dr. KELLY's statements be exact, as there is no doubt they are, and Messrs. BINGLEY's and VANDOME's pound be *really* the two remaining weights of the often-mentioned three which Mr. HARRIS presented to the Committee of 1758*, we can still either determine, with the highest degree of probability, the specific gravity of the lost imperial standard troy pound, or know with certainty that all hope to arrive at this knowledge is lost. It will be only requisite to ascertain with the greatest care the specific gravities of both pounds, the one in the possession of Mr. BINGLEY, and the other in the possession of Mr. VANDOME. If the specific gravity of both is found *the*

* There is an easy and obvious verification of the fact. If Mr. BINGLEY's and Mr. VANDOME's pounds be indeed the two remaining pounds of 1758, they must have the *same* stamps which the standard had, of which an exact representation, taken by Captain NEHUS in 1829, is subjoined to this paper. The Report of the Committee of 1758 says expressly (p. 437 b.), "Three several troy pounds to be made under the direction of "Mr. HARRIS, to be *marked* as follows:" here the representation of the stamps of the standard roughly cut in wood is added, which, according to the words of the Report, must be common to them all.

same, we might from that circumstance draw the highly probable conclusion, that the three single pounds of Mr. Harris, according to my hypothesis, were really made of the same identical metal; and the specific gravity of the two remaining pounds might with safety be considered as that of the lost standard. If, on the contrary, the two remaining pounds prove to be of *different* specific gravities, the hypothesis that all three were made of the same metal is evidently erroneous; and nothing can be inferred from the specific gravity of either of the two remaining. For in this case the metal of the lost standard may have been,

1. Identical with that of Mr. Bingley's pound, or
2. Identical with that of Mr. Vandome's pound, or
3. A different metal from that of both these pounds.

Now as there is no metal of which we know, except that of the two remaining pounds, that may be considered as identical with that of the lost standard, it is evident that if this also cannot be considered as being so, all hope is lost of arriving at the knowledge of the specific gravity of the late imperial standard troy pound.

27. It may be worth while to express in numbers the uncertainty that remains about the *true* weight of the lost standard pound as the case now stands; that is, without knowing if it was of brass or copper, and without having a precise knowledge of its specific gravity. To do this it will be necessary to state the formulæ and the numeric values used by me in the computation of its true difference of weight with all the pounds that have been compared with it; or, what is the same, in the reduction to a vacuum of all the weighings made. I have adopted both the formulæ and the numeric values contained in M. Bessel's excellent paper on the reduction of weighings in the *Astron. Nachrichten*, vol. vii. p. 373.

The specific gravity of a body is the quotient of its density divided by the density of that substance, which is considered as unity: as such, pure water is here adopted. But since both these densities vary with the temperature,—because the same invariable quantity of matter which the body contains is always distributed over its volume, variable with the temperature, so that generally speaking (the exception which pure water affords will immediately be noticed) the body has at a higher temperature less density than at a lower,—we must fix a certain temperature at which the body as well as the water is to be considered. It is not necessary that this fixed temperature should be the same for the body and the water, its choice for both being quite arbitrary.

For bodies, the most natural seems to be that of one of the fixed points of the thermometer; and the temperature of melting ice (Fahrenheit 32°, Réaumur and Centigrade 0°) is here adopted. For pure water, it is known that there is a maximum of its density, which takes place at nearly 39° Fahr.; and this maximum of density, or the density of pure water under the temperature of nearly 39° Fahr., is by preference adopted as unity.

Now as the densities of two bodies are in the direct ratio of their masses, and in the inverse ratio of their volumes, we can express the specific gravity of a body as the quotient of its mass, divided by the mass of pure water taken at its greatest density contained in a volume equal to that which the body occupies at 32° FAHR.: or, what is the same, as the quotient of its mass, divided by the mass of pure water which the body displaces; the water having the temperature of nearly 39° FAHR., and the body that of 32° FAHR.

If we denote the specific gravity of the body, thus understood, by Δ, the mass of the body by M, and the ratio of one of its dimensions under the temperature of the melting ice, and under that of the weighing, by ... 1 : R, the space which it occupies at the temperature which it has when it is weighed is $= \frac{M}{\Delta} R^3$, and it displaces a mass of air equal to $\frac{M}{\Delta} R^3 q$, where q denotes the specific gravity of the air at the moment of the weighing.

For the weights employed, let δ, m, r denote the same things as Δ, M, R for the body. We have consequently, when the body and the weights put upon the balance are in equilibrio, the equation

$$M\left(1 - \frac{R^3 q}{\Delta}\right) = m\left(1 - \frac{r^3 q}{\delta}\right) \tag{1}$$

whereby M, or the absolute weight of the body, is easily obtained, viz.

$$M = m + M\frac{R^3 q}{\Delta} - m\frac{r^3 q}{\delta} \tag{2}$$

or, as m may in the second number of the equation be generally substituted for M,

$$M = m + m\frac{R^3 q}{\Delta} - m\frac{r^3 q}{\delta} \text{ nearly.} \tag{3}$$

Should a case occur in which this substitution would affect the last place of decimals, we may employ, either the exact equation, derived immediately from (1)

$$M = m\,\frac{1 - \frac{r^3 q}{\delta}}{1 - \frac{R^3 q}{\Delta}} \tag{4}$$

or put the value of M, found by the equation (3) as coefficient of $\frac{R^3 q}{\Delta}$, into the equation (2).

28. In order to obtain the specific gravity, the body is weighed in air, and also when immersed in pure water. In the last case, as in the former, the *weights* are still in air. These two operations give, if we denote by

Q . . . the specific gravity of the water * at the temperature which it has when the body is immersed in it, and by

m', r', q', R' . . . the values of m, r, q, R at the weighing in water †,

the following two equations, viz.

$$\text{for weighing in air,} \quad M\left(1-\frac{R^3 q}{\Delta}\right)=m\left(1-\frac{r^3 q}{\delta}\right) \tag{1}$$

$$\text{for weighing in water,} \quad M\left(1-\frac{R'^3 Q}{\Delta}\right)=m'\left(1-\frac{r'^3 q'}{\delta}\right) \tag{5}$$

whence, by eliminating M, we obtain

$$\Delta=\frac{m\,R'^3 Q\left(1-\frac{r^3 q}{\delta}\right)-m'\,R^3 q\left(1-\frac{r'^3 q'}{\delta}\right)}{m\left(1-\frac{r^3 q}{\delta}\right)-m'\left(1-\frac{r'^3 q'}{\delta}\right)} \tag{6}$$

or, if for brevity's sake we put $\frac{r^3 q}{\delta}=a, \quad \frac{r'^3 q'}{\delta}=a'$,

$$\Delta=\frac{m\,R'^3 Q\,(1-a)-m'\,R^3 q\,(1-a')}{m\,(1-a)-m'\,(1-a')} \tag{7}‡$$

If only the first power of a is taken into consideration, which (with the exception of elastic fluids) can cause no perceptible error, we have the approximate formula

$$\Delta=\frac{m}{m-m'}R'^3 Q-\frac{m'}{m-m'}R^3 q+\frac{m\,m'}{(m-m')^2}\left(R'^3 Q-R^3 q\right)(a-a') \tag{8}$$

or, because R and R′ are nearly equal to 1, and q so small that it may be neglected, we may put $R'^3 Q-R^3 q=Q$, and obtain

$$\Delta=\frac{m}{m-m'}R'^3 Q-\frac{m'}{m-m'}R^3 q+\frac{m\,m'}{(m-m')^2}Q\,(a-a') \tag{9}‡$$

29. It remains now to determine the numeric values to be used for these reductions, and to give Tables that make the application of the formulæ more easy.

* The unity adopted for specific gravities being pure water at its maximum of density, Q is of course the density of pure water at the temperature T, divided by its greatest density, or

$$Q=\frac{\text{density of pure water at the temperature T.}}{\text{density of pure water at the temperature of nearly } 39^\circ \text{ F.}}$$

T is the common temperature of the water in which the body is immersed, and of the body immersed in it.

† The weight of the body immersed in water (m') is evidently different from its weight in air (m), and the temperatures of the water and of the air, at the moment when the body is weighed in water, will generally be different from the temperature of the air for the moment when the body is weighed in air. The values of r, q, R depend on these temperatures, and will consequently generally be different in both cases, so that they ought to be distinguished by a particular notation with accents: r', q' depends on t' (= common temperature of the air and the weights when the body is weighed in water), and R′ depends on T (= common temperature of the water and the body immersed in it).

‡ When the atmospheric circumstances are the *same* at the weighing in water as they were at the weighing in air, or, in other words, if $b'=b$, and $t'=t$, it is not necessary to know the specific gravity of the weights em-

M. BESSEL supposes that atmospheric air, at the temperature of melting ice, and under the pressure of 29·922 English inches (= 0·76 metre) of mercury, has the specific gravity of

$$\frac{13{\cdot}59606}{10475{\cdot}6} = \frac{1}{770{\cdot}488}$$

where the numerator is the specific gravity of mercury according to BRISSON's experiments calculated by HÄLLSTRÖM, and the denominator the ratio of the density of air to that of mercury found by MM. BIOT and ARAGO. This gives for q, or for the height of the barometer expressed in English inches and reduced to the density of mercury at the temperature of melting ice, and for t, the temperature of the air expressed in FAHRENHEIT degrees,

$$q = b \cdot \frac{1}{770{\cdot}488} \cdot \frac{1}{29{\cdot}922} \cdot \frac{1}{1 + (t - 32^\circ)\, 0{\cdot}0020833} = b \cdot \frac{1}{23054{\cdot}39[1 + (t - 32^\circ)\, 0{\cdot}0020833]} \qquad (10)$$

Supposing weights of brass whose specific gravity = 8 (the correction for the actual specific gravity of the brass weights differing from 8 is easily applied, as will immediately be shown), and taking the linear expansion of brass for one degree of FAHRENHEIT's scale = 0·000010436, we have for this metal

$$r^3 = [1 + (t - 32^\circ)\, 0{\cdot}000010436]^3$$

and consequently,

$$\frac{r^3 q}{\delta} = a = b \cdot \frac{[1 + (t - 32^\circ)\, 0{\cdot}000010436]^3}{23054{\cdot}39\,[1 + (t - 32^\circ)\, 0{\cdot}0020833]} \cdot \frac{1}{8}$$

Table I., here following, contains the logarithm of the coefficient of b in this formula, which coefficient we shall denote by α; so that $a = b\,\alpha$; the argument of which is the temperature of the air (the temperature of the weights being supposed equal to that of the air), or t in FAHRENHEIT degrees. If the body is weighed in water, it is evident that α' must be taken with the argument t' (= temperature of

ployed for that purpose, nor even of what metal they are. Indeed, a depends on b and t, and a' on b' and t'; consequently if $b' = b$, and $t' = t$, we have also $a' = a$, and the fraction $\frac{m\,R'^3\,Q\,(1 - a) - m'\,R^3\,q\,(1 - a')}{m\,(1 - a) - m'\,(1 - a')}$ has the common factor in the denominator, and numerator $(1 - a)$, which consequently disappears, and reduces it to

$$\Delta = \frac{m\,R'^3\,Q}{m - m'} - \frac{m'\,R^3\,q}{m - m'}$$

The same result is obtained by the equation (9), in which in this case

$$\frac{m\,m'}{(m - m')^2} \cdot Q\,(a - a') = 0$$

so that we obtain as before

$$\Delta = \frac{m\,R'^3\,Q}{m - m'} - \frac{m'\,R^3\,q}{m - m'}$$

If the atmospheric circumstances are nearly the same in both weighings, the precise knowledge of the specific gravity of the weights employed has little influence, and always less in proportion as b' is nearer to b, and t' nearer to t.

the air at that moment), and that b', or the height of the barometer when the body is weighed in water, must be employed. We thus obtain

$$\frac{r'^3 q'}{\delta} = a' = b' \alpha'$$

If the brass, of which the weights are made, has a specific gravity $= \delta$, different from that assumed $(= 8)$, α must be first multiplied by 8, and afterwards divided by δ. This comes to the same as applying to the numbers of Table I. the correction c; c being $= \log 8 - \log \delta$, or $c = 0{\cdot}90309 - \log \delta$.

30. We come now to $\frac{R^3 q}{\Delta}$. If the linear expansion of the weighed body for one degree of Fahrenheit's scale be denoted by e, we have $R^3 = [1 + (t - 32^\circ)\, e]^3$. For q we have already the equation (10), in which, by making

$$\beta = \frac{1}{23054{\cdot}39 \,.\, [1 + (t - 32^\circ)\, 0{\cdot}0020833]}$$

we have $q = b\,\beta$: β is evidently the specific gravity of atmospheric air at the temperature t, and under the pressure of one English inch of mercury, the mercury being reduced to its density at the temperature of the melting ice. We thus obtain

$$\frac{R^3 q}{\Delta} = b\,\beta \,.\, \frac{[1 + (t - 32^\circ)\, e]^3}{\Delta}.$$

Table II. contains the logarithms of β. Its argument is the temperature of the air in Fahrenheit degrees at the moment of the weighing.

Tables III. IV. V. contain the logarithms of $[1 + (t - 32^\circ)\, e]^3$ for brass, copper, and platina. They suppose

For brass $e = 0{\cdot}000010436$
For copper $e = 0{\cdot}000009541$
For platina $e = 0{\cdot}000005000$ and $0{\cdot}000005050$

Table I.—For Brass weights the specific gravity of which = 8.

t.	log α.		t.	log α.		t.	log α.		t.	log α.		t.	log α.		t.	log α.	
32°	4·73416	89	41°	4·72621	87	50°	4·71841	85	59°	4·71076	85	68°	4·70324	83	77°	4·69585	81
33	4·73327	89	42	4·72534	87	51	4·71756	86	60	4·70991	84	69	4·70241	82	78	4·69504	81
34	4·73238	89	43	4·72447	87	52	4·71670	85	61	4·70907	83	70	4·70159	83	79	4·69423	81
35	4·73149	88	44	4·72360	87	53	4·71585	86	62	4·70824	84	71	4·70076	82	80	4·69342	81
36	4·73061	89	45	4·72273	87	54	4·71499	85	63	4·70740	84	72	4·69994	82	81	4·69261	81
37	4·72972	88	46	4·72186	86	55	4·71414	85	64	4·70656	83	73	4·69912	82	82	4·69180	80
38	4·72884	88	47	4·72100	87	56	4·71329	84	65	4·70573	83	74	4·69830	82	83	4·69100	81
39	4·72796	87	48	4·72013	86	57	4·71245	85	66	4·70490	83	75	4·69748	81	84	4·69019	80
40	4·72709	88	49	4·71927	86	58	4·71160	84	67	4·70407	83	76	4·69667	82	85	4·68939	80
41	4·72621		50	4·71841		59	4·71076		68	4·70324		77	4·69585		86	4·68859	

Log α is taken with the temperature of the air in Fahrenheit degrees at the moment of the weighing. If the brass weights are of any other specific gravity $= \delta$, a correction c must be applied to the numbers of the table: or $c = 0{\cdot}90309 - \log \delta$.

Table II.—Containing the Logarithms of β.

t.	log β.		*t.*	log β.		*t.*	log β.		*t.*	log β.		*t.*	log β.		*t.*	log β.	
32°	5·63725		41°	5·62918		50°	5·62126		59°	5·61348		68°	5·60584		77°	5·59833	
33	5·63634	91	42	5·62829	89	51	5·62039	87	60	5·61262	86	69	5·60500	84	78	5·59750	83
34	5·63544	90	43	5·62741	88	52	5·61952	87	61	5·61177	85	70	5·60416	84	79	5·59668	82
35	5·63454	90	44	5·62652	89	53	5·61865	87	62	5.61092	85	71	5·60332	84	80	5·59585	83
36	5·63364	90	45	5·62564	88	54	5·61778	87	63	5·61007	85	72	5·60248	84	81	5·59503	82
37	5·63275	89	46	5·62476	88	55	5·61692	86	64	5·60922	85	73	5·60165	83	82	5·59421	82
38	5·63185	90	47	5·62388	88	56	5·61606	86	65	5·60837	85	74	5·60082	83	83	5·59339	82
39	5·63096	89	48	5·62301	87	57	5·61520	86	66	5·60752	85	75	5·59999	83	84	5·59258	81
40	5·63007	89	49	5·62213	88	58	5·61434	86	67	5·60668	84	76	5·59916	83	85	5·59176	82
41	5·62918	89	50	5·62126	87	59	5·61348	86	68	5·60584	84	77	5·59833	83	86	5·59085	81

Log β is taken with the temperature of the air in Fahrenheit degrees at the moment of the weighing.

Table III.—Containing the Logarithms of $R^3 = [1 + (t - 32°)\, e]^3$ for *Brass*, assuming $e = 0·000010436$.

t.	log R^3.	*t.*	log R^3.	*t.*	log R^3.	*t.*	log R^3.	*t.*	log R^3.
32°	0·0000000	43°	0·0001496	54°	0·0002991	65°	0·0004486	76°	0·0005981
33	0·0000136	44	0·0001632	55	0·0003127	66	0·0004622	77	0·0006117
34	0·0000272	45	0·0001768	56	0·0003263	67	0·0004758	78	0·0006253
35	0·0000408	46	0·0001904	57	0·0003399	68	0·0004894	79	0·0006389
36	0·0000544	47	0·0002040	58	0·0003535	69	0·0005030	80	0·0006525
37	0·0000680	48	0·0002176	59	0·0003670	70	0·0005166	81	0·0006661
38	0·0000816	49	0·0002311	60	0·0003806	71	0·0005302	82	0·0006796
39	0·0000952	50	0·0002447	61	0·0003942	72	0·0005438	83	0·0006932
40	0·0001088	51	0·0002583	62	0·0004078	73	0·0005673	84	0·0007068
41	0·0001224	52	0·0002719	63	0·0004214	74	0·0005709	85	0·0007204
42	0·0001360	53	0·0002855	64	0·0004350	75	0·0005845	86	0·0007340

Proportional parts.	
0°·1	14
0·2	27
0·3	41
0·4	54
0·5	68
0·6	82
0·7	95
0·8	109
0·9	122

Log R^3 is taken with the temperature of the air in Fahrenheit degrees when the body is in the air, and with the temperature of the water when the body is immersed in water.

Table IV.—Containing the Logarithms of $R^3 = [1 + (t - 32)\, e]^3$ for *Copper*, assuming $e = 0·000009541$.

t.	log R^3.	*t.*	log R^3.	*t.*	log R^3.	*t.*	log R^3.	*t.*	log R^3.
32°	0·0000000	43°	0·0001367	54°	0·0002734	65°	0·0004102	76°	0·0005468
33	0·0000124	44	0·0001491	55	0·0002859	66	0·0004226	77	0·0005593
34	0·0000249	45	0·0001616	56	0·0002983	67	0·0004350	78	0·0005717
35	0·0000373	46	0·0001740	57	0·0003107	68	0·0004474	79	0·0005841
36	0·0000497	47	0·0001864	58	0·0003232	69	0·0004599	80	0·0005966
37	0·0000621	48	0·0001989	59	0·0003356	70	0·0004723	81	0·0006090
38	0·0000746	49	0·0002113	60	0·0003480	71	0·0004847	82	0·0006214
39	0·0000870	50	0·0002237	61	0·0003608	72	0·0004971	83	0·0006338
40	0·0000994	51	0·0002361	62	0·0003729	73	0·0005096	84	0·0006462
41	0·0001119	52	0·0002486	63	0·0003853	74	0·0005220	85	0·0006587
42	0·0001243	53	0·0002610	64	0·0003977	75	0·0005344	86	0·0006711

Proportional parts.	
0°·1	12
0·2	25
0·3	37
0·4	50
0·5	62
0·6	75
0·7	87
0·8	99
0·9	112

Log R^3 is taken with the temperature of the air in Fahrenheit degrees when the body is in the air, and with the temperature of the water when it is immersed in water.

TABLE V.—Containing the Logarithms of $R^3 = [1 + (t-32^\circ)\,e]^3$ for *Platina*.

Assuming $e = 0{\cdot}000005000$.

t.	log R³.	t.	log R³.
32°	0·0000000	60°	0·0001824
33	0·0000065	61	0·0001889
34	0·0000130	62	0·0001954
35	0·0000195	63	0·0002019
36	0·0000261	64	0·0002085
37	0·0000326	65	0·0002150
38	0·0000391	66	0·0002215
39	0·0000456	67	0·0002280
40	0·0000521	68	0·0002345
41	0·0000586	69	0·0002410
42	0·0000651	70	0·0002475
43	0·0000717	71	0·0002540
44	0·0000782	72	0·0002606
45	0·0000847	73	0·0002671
46	0·0000912	74	0·0002736
47	0·0000977	75	0·0002801
48	0·0001042	76	0·0002866
49	0·0001107	77	0·0002931
50	0·0001173	78	0·0002996
51	0·0001238	79	0·0003061
52	0·0001303	80	0·0003127
53	0·0001368	81	0·0003192
54	0·0001433	82	0·0003257
55	0·0001498	83	0·0003322
56	0·0001563	84	0·0003387
57	0·0001629	85	0·0003452
58	0·0001693	86	0·0003517
59	0·0001759		

Assuming $e = 0{\cdot}000005050$.

t.	log R³.	t.	log R³.
32°	0·0000000	60°	0·0001842
33	0·0000066	61	0·0001908
34	0·0000132	62	0·0001974
35	0·0000197	63	0·0002040
36	0·0000263	64	0·0002106
37	0·0000329	65	0·0002171
38	0·0000395	66	0·0002237
39	0·0000461	67	0·0002303
40	0·0000526	68	0·0002369
41	0·0000592	69	0·0002435
42	0·0000658	70	0·0002500
43	0·0000724	71	0·0002566
44	0·0000790	72	0·0002632
45	0·0000855	73	0·0002698
46	0·0000921	74	0·0002764
47	0·0000987	75	0·0002829
48	0·0001053	76	0·0002895
49	0·0001119	77	0·0002961
50	0·0001184	78	0·0003027
51	0·0001250	79	0·0003093
52	0·0001316	80	0·0003158
53	0·0001382	81	0·0003224
54	0·0001448	82	0·0003289
55	0·0001513	83	0·0003355
56	0·0001579	84	0·0003421
57	0·0001645	85	0·0003486
58	0·0001711	86	0·0003552
59	0·0001777		

Proportional parts.

0°·1	7
0·2	13
0·3	20
0·4	26
0·5	33
0·6	40
0·7	46
0·8	53
0·9	59

Log R^3 is taken with the temperature of the air in FAHRENHEIT degrees when the body is in the air, and with the temperature of the water when the body is immersed in water.

31. We shall add to these tables a Table for the logarithms of Q, which is used when, by weighing in water, the specific gravity of a body ($=\Delta$) is to be determined. The table M. BESSEL has given is that of HÄLLSTRÖM, the result of his experiments and calculations published in *Vetenskaps Academiens Handlingar* för år 1823, and reprinted in POGGENDORFF'S *Annalen der Physik*, vol. i. p. 168. HÄLLSTRÖM found the density of pure water, between the limits of 0° and +32° of the centigrade thermometer,

$$= 1 + 0{\cdot}000052939\,\tau - 0{\cdot}0000065322\,\tau^2 + 0{\cdot}00000001445\,\tau^3$$

where τ denotes the degrees of the centigrade thermometer. The unity is here the density of pure water at the temperature of melting ice. This formula gives the greatest density of pure water $= 1{\cdot}00010824$, and the temperature at which it occurs $= +\,4^\circ{\cdot}108$ centigrade. We have consequently,

$$Q = \frac{1}{1{\cdot}00010824}\,(1 + 0{\cdot}000052939\,\tau - 0{\cdot}0000065322\,\tau^2 + 0{\cdot}0000000144\,\tau^3)$$

and the logarithms of these values of Q are given by M. Bessel in the *Astron. Nach.*, vol. vii. p. 376.

Ten years later M. Hällström resumed the subject, and adding to his experiments those of MM. Muncke and Stampfer, made since his first determination, gave a new Table for the density of pure water in the *Vetenskaps Academiens Handlingar* för år 1833. The whole paper is translated in Poggendorff's *Annalen der Physik*, vol. xxxiv. p. 220 et seq. M. Hällström finds for the *volume* of pure water between the limits of 0° and + 30° of the centigrade thermometer*,

$$1 - 0{\cdot}000057590\,\tau + 0{\cdot}0000075611\,\tau^2 - 0{\cdot}000000035100\,\tau^3$$

where τ denotes degrees of the centigrade thermometer; and where the volume of pure water at the temperature of melting ice is considered as unity. If we call the volume ν, we obtain hence, for Fahrenheit's degrees (denoted by t), the formula

$$\nu = 1 - 0{\cdot}0000319945\,(t-32^\circ) + 0{\cdot}00000233367\,(t-32^\circ)^2 - 0{\cdot}00000000601848\,(t-32^\circ)^3,$$

which gives the minimum of volume (for $t = 39^\circ{\cdot}047$) $= 0{\cdot}99988832$, and consequently the maximum of density $= 1{\cdot}0001117$. Now the density being $= \frac{1}{\nu}$, we obtain

$$Q = \frac{1}{1{\cdot}0001117.(1 - 0{\cdot}0000319945\,(t-32) + 0{\cdot}00000233367\,(t-32)^2 - 0{\cdot}00000000601848\,(t-32)^3)}.$$

Agreeably to this equation the values of Q, whose logarithms are given in the following Table, are calculated; which Table contains also the values of ν as above stated, as well as of D, together with their logarithms: D being the density of pure water at the temperature t (in Fahrenheit degrees), the density at 32° being = 1.

* There are errors of the press, or oversights in calculation, in the original memoir of M. Hällström, repeated in the translation, which I have corrected here. The equation for ν should be the arithmetic mean (Poggendorff, p. 246.) of the four equations which M. Hällström calls I. V. VI. IX. Now we have

(I.) p. 228. $\nu = 1 - 0{\cdot}000049976\,\tau + 0{\cdot}0000062453\,\tau^2 - 0{\cdot}000000007645\,\tau^3$

(V.) p. 238. $\nu = 1 - 0{\cdot}000060835\,\tau + 0{\cdot}0000081037\,\tau^2 - 0{\cdot}000000048282\,\tau^3$

(VI.) p. 239. $\nu = 1 - 0{\cdot}000059269\,\tau + 0{\cdot}0000076816\,\tau^2 - 0{\cdot}000000037159\,\tau^3$

(IX.) p. 244. $\nu = 1 - 0{\cdot}000060280\,\tau + 0{\cdot}0000082138\,\tau^2 - 0{\cdot}000000047313\,\tau^3$

The arithmetic mean of these four equations is

$$\nu = 1 - 0{\cdot}000057590\,\tau + 0{\cdot}0000075611\,\tau^2 - 0{\cdot}000000035100\,\tau^3,$$

as above stated, and not

$$\nu = 1 - 0{\cdot}000057577\,\tau + 0{\cdot}0000075601\,\tau^2 - 0{\cdot}000000035091\,\tau^3,$$

as M. Hällström has it. The Table (p. 247.) likewise which he has calculated upon his formula for ν and D, has several inaccuracies.

Table VI.—Containing the logarithms of Q; and also the values of ν and D, and their logarithms.

t.	log Q.		ν.		log ν.		D.		log D.	
32°	9·9999515		1·0000000		0·0000000		1·0000000		0·0000000	
33	9·9999644	+129	0·9999703	−297	9·9999871	−129	1·0000297	+297	0·0000129	+129
34	9·9999753	109	0·9999453	250	9·9999762	109	1·0000547	250	0·0000238	109
35	9·9999841	88	0·9999249	204	9·9999674	88	1·0000751	204	0·0000326	88
36	9·9999911	70	0·9999090	159	9·9999604	70	1·0000910	159	0·0000396	70
37	9·9999960	49	0·9998976	114	9·9999555	49	1·0001024	114	0·0000445	49
38	9·9999990	30	0·9998907	69	9·9999525	30	1·0001093	69	0·0000475	30
39	0·0000000	+10	0·9998883	−24	9·9999515	−10	1·0001117	+24	0·0000485	+10
40	9·9999991	−9	0·9998903	+20	9·9999524	+9	1·0001097	−20	0·0000476	−9
41	9·9999964	27	0·9998967	64	9·9999551	27	1.0001033	64	0·0000449	27
42	9·9999917	47	0·9999074	107	9·9999598	47	1·0000926	107	0·0000402	47
43	9·9999852	65	0·9999224	150	9·9999663	65	1·0000776	150	0·0000337	65
44	9·9999768	84	0·9999417	193	9·9999747	84	1·0000583	193	0·0000253	84
45	9·9999666	102	0·9999652	235	9·9999849	102	1·0000348	235	0·0000151	102
46	9·9999545	121	0·9999930	278	9·9999970	121	1·0000070	278	0·0000030	121
47	9·9999407	138	1·0000249	319	0·0000108	138	0·9999751	319	9·9999892	138
48	9·9999250	157	1·0000609	360	0·0000265	157	0.9999391	360	9·9999735	157
49	9·9999077	173	1·0001009	400	0·0000438	173	0·9998991	400	9·9999562	173
50	9·9998885	192	1·0001451	442	0·0000630	192	0·9998549	442	9·9999370	192
51	9·9998676	209	1·0001933	482	0·0000839	209	0·9998067	482	9·9999161	209
52	9·9998449	227	1·0002454	521	0·0001066	227	0·9997547	520	9·9998934	227
53	9·9998206	243	1·0003015	561	0·0001309	243	0·9996986	561	9·9998691	243
54	9·9997945	261	1·0003615	600	0·0001570	261	0·9996386	600	9·9998430	261
55	9·9997668	277	1·0004254	639	0·0001847	277	0·9995748	638	9·9998153	277
56	9·9997374	294	1·0004931	677	0·0002141	294	0·9995071	677	9·9997859	294
57	9·9997064	310	1·0005646	715	0·0002451	310	0·9994357	714	9·9997549	310
58	9·9996737	327	1·0006399	753	0·0002778	327	0·9993605	752	9·9997222	327
59	9·9996394	343	1·0007189	790	0·0003121	343	0·9992816	789	9·9996879	343
60	9·9996035	359	1·0008016	827	0·0003480	359	0·9991990	826	9·9996520	359
61	9·9995660	375	1·0008880	864	0·0003855	375	0·9991128	862	9·9996145	375
62	9·9995270	390	1·0009780	900	0·0004245	390	0·9990230	898	9·9995755	390
63	9·9994864	406	1·0010715	935	0·0004651	406	0·9989296	934	9·9995349	406
64	9·9994443	421	1·0011686	971	0·0005072	421	0·9988328	968	9·9994928	421
65	9·9994006	437	1·0012693	1007	0·0005509	437	0·9987323	1005	9·9994491	437
66	9·9993554	452	1·0013734	1041	0·0005961	452	0·9986285	1038	9·9994039	452
67	9·9993088	466	1·0014809	1075	0·0006427	466	0·9985213	1072	9·9993573	466
68	9·9992607	481	1·0015918	1109	0·0006908	481	0·9984107	1106	9·9993092	481
69	9·9992112	495	1·0017061	1143	0·0007403	495	0·9982968	1139	9·9992597	495
70	9·9991602	510	1·0018238	1177	0·0007913	510	0·9981795	1173	9·9992087	510
71	9·9991077	525	1·0019447	1209	0·0008438	525	0·9980591	1204	9·9991562	525
72	9·9990539	538	1·0020689	1242	0·0008976	538	0·9979354	1237	9·9991024	538
73	9·9989987	552	1·0021963	1274	0·0009528	552	0·9978085	1269	9·9990472	552
74	9·9989421	566	1·0023269	1306	0·0010094	566	0·9976785	1300	9·9989906	566
75	9·9988841	580	1·0024607	1338	0·0010674	580	0·9975454	1331	9·9989326	580
76	9·9988248	593	1·0025976	1369	0·0011267	593	0·9974091	1363	9·9988733	593
77	9·9987642	606	1·0027375	1399	0·0011873	606	0·9972700	1391	9·9988127	606
78	9·9987023	619	1·0028805	1430	0·0012492	619	0·9971278	1422	9·9987508	619
79	9·9986391	632	1·0030265	1460	0·0013124	632	0·9969826	1452	9·9986876	632
80	9·9985746	645	1·0031755	1490	0·0013769	645	0·9968346	1480	9·9986231	645
81	9·9985088	658	1.0033274	1519	0·0014427	658	0·9966836	1510	9·9985573	658
82	9·9984418	670	1·0034822	1548	0·0015097	670	0·9965299	1537	9·9984903	670
83	9·9983736	682	1·0036398	1576	0·0015779	682	0·9963734	1565	9·9984221	682
84	9·9983042	694	1·0038003	1605	0·0016473	694	0·9962141	1593	9·9983527	694
85	9·9982335	707	1·0039636	1633	0·0017180	707	0·9960521	1620	9·9982820	707
86	9·9981617	−718	1·0041296	+1660	0·0017898	+718	0·9958874	−1648	9·9982102	−718

32. After having given all the elements and formulæ necessary for the reduction of the weighings, we may now state in numbers the uncertainty that remains about the true weight of the lost standard pound; respecting which at this moment we do not know whether it was made of brass or of copper (though the probability inclines

for brass), and of whose specific gravity, even if we assume it of brass, we are equally ignorant.

We will previously state the following results of the weighings given in § 19. in a form adapted for the subsequent calculations.

m.	log *b.*	*t.*	Δ.	$\log \frac{1}{\Delta}$.	*c.*
K = 5760·03389 gr.	1·47197	65°·09	7·994	......	+0·00033
S.p = 5759·99143	1·47308	65·62	21·1874	8·67392	
S.b = 5759·98966	1·47661	64·50	8·228	......	−0·01220
RS = 5759·99795	1·47430	65·73	21·1874	8·67392	
RM = 5760·00887	1·47245	65·91	7·994	......	+0·00033

The column marked with Δ contains the specific gravities of the pounds compared with U. Of these, the specific gravities of K and S.p only have been found by weighing in water*. The specific gravity of S.b is the mean of the specific gravities of two other brass weights made by Mr. Robinson, supposing that his brass was always nearly of the same specific gravity, which is indeed a precarious supposition, but may be adopted until S.b itself be weighed in water. The specific gravities of RS and RM are unknown. I have, until they be determined, supposed RS of the same specific gravity with S.p, and RM of the same specific gravity with K. Indeed K and RM are both made by Mr. Bate, and probably at the same time, so that it seems allowable to suppose that they are of the same metal.

The column *c* contains the correction for the three brass pounds to be applied to the log *α* found in Table I., on account of their specific gravities differing from 8·0, which is the specific gravity supposed in Table I. This correction is = 0·90309 − log δ (see § 29.).

33. We may now calculate the reduction to a vacuum for these weighings, on two hypotheses; assuming U,

1°. to have been of brass, with the specific gravity = 8·0;

2°. to have been of copper, with the specific gravity = 8·788.

In the first hypothesis, the logarithm of the quantity $\beta \frac{r^3}{\delta} = \alpha$, may for U be taken immediately out of Table I. Likewise the log of the quantity $\beta \frac{R^3}{\Delta}$ may for the three brass pounds be taken out of Table I., on applying to the log *α* the correction *c*, stated at the bottom of that Table. For the two platina pounds, $\beta \frac{R^3}{\Delta}$ must be calculated separately, which is also the case with $\beta \frac{r^3}{\delta}$ if we assume U to have been of copper. Log *β* is taken out of Table II.

* S.p itself has not yet been weighed in water. Its specific gravity is determined by weighing in water another weight of the same platina, which Mr. Robinson made for me, for that purpose.

The formula (3) becomes, after having put for q its value $b\,\beta$,

$$M = m + m\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta},$$

which, it must be remembered, is only an approximate formula, the exact formula being

$$M = m + M\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta}.$$

In general the logarithm of $m\,b\,\beta\frac{R^3}{\Delta}$ will be identical with the logarithm of $M\,b\,\beta\frac{R^3}{\Delta}$ when we use logarithms with five decimals, which give even more accuracy than the weighings can pretend to: but should log M (M being found by the first equation) differ in the fifth decimal from log m, we must use the value of M obtained by the first equation and put it in the latter, in the term $M\,b\,\beta\frac{R^3}{\Delta}$, in order to obtain a result for M as exact as may be found with logarithms of five decimals.

Reduction of the weighings of K.

U supposed of Brass.

$\log m = 3{\cdot}76042$	$\log m = 3{\cdot}76042$
$\log b = 1{\cdot}47197$	$\log b = 1{\cdot}47197$
(Table I.) $\log \alpha = 4{\cdot}70566$	$\log \alpha = 4{\cdot}70566$
$c = 0{\cdot}00033$	$9{\cdot}93805$
$9{\cdot}93838$	$m\,b\,\alpha = 0{\cdot}86706$
$m\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}86772$	

$$m\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\alpha = 0{\cdot}86772 - 0{\cdot}86706 = +0{\cdot}00066$$

$m = 5760{\cdot}03389$

$M = 5760{\cdot}03455$

The logarithm of M is 3·76042, the same as that of m, so that it is not necessary to repeat the calculation with log M.

U supposed of Copper.

$m\,b\,\beta\frac{R^3}{\Delta}$ as before $= 0{\cdot}86772$

$\log m$	$= 3{\cdot}76042$
$\log b$	$= 1{\cdot}47197$
(Table II.) $\log \beta$	$= 5{\cdot}60829$
(Table IV.) $\log r^3$	$= 0{\cdot}00041$
$\log\frac{1}{\delta}$	$= 9{\cdot}05611$
	$9{\cdot}89720$
$m\,b\,\beta\frac{r^3}{\delta}$	$= 0{\cdot}78922$

$$m\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}86772 - 0{\cdot}78922 = +0{\cdot}07850$$

$m = 5760{\cdot}03389$

$M = 5760{\cdot}11239$ nearly.

The logarithm of M is 3·76043, being by one unity in the fifth decimal greater than log m. Therefore, by using log M instead of log m, we obtain

$\log M\,b\,\beta\frac{R^3}{\Delta} = 9{\cdot}93839$ $\quad$ $M\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}86774$

$$M\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}86774 - 0{\cdot}78922 = +0{\cdot}07852$$

$m = 5760{\cdot}03389$

Correct value of $M = 5760{\cdot}11241$

Reduction of the weighings of S.p.

U supposed of Brass.

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47308$

(Tab. II.) $\log \beta = 5{\cdot}60784$

(Tab. V.) $\log R^3 = 0{\cdot}00022$

$\log \frac{1}{\Delta} = 8{\cdot}67392$

$9{\cdot}51548$

$m\,b\,\beta \frac{R^3}{\Delta} = 0{\cdot}32770$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47308$

(Tab. I.) $\log \alpha = 4{\cdot}70522$

$9{\cdot}93872$

$m\,b\,\alpha = 0{\cdot}86840$

$$m\,b\,\beta \frac{R^3}{\Delta} - m\,b\,\alpha = 0{\cdot}32770 - 0{\cdot}86840 = -0{\cdot}54070$$

$m = 5759{\cdot}99143$

$M = 5759{\cdot}45073$ nearly.

The logarithm of M is 3·76038, four unities in the fifth decimal less than log m. We obtain by using, as before, log M instead of log m,

$\log M\,b\,\beta \frac{R^3}{\Delta} = 9{\cdot}51544$ $\quad M\,b\,\beta \frac{R^3}{\Delta} = 0{\cdot}32767$

$$M\,b\,\beta \frac{R^3}{\Delta} - m\,b\,\alpha = 0{\cdot}32767 - 0{\cdot}86840 = -0{\cdot}54073$$

$m = 5759{\cdot}99143$

Correct value of $M = 5759{\cdot}45070$

U supposed of Copper.

$m\,b\,\beta \frac{R^3}{\Delta}$ as before $= 0{\cdot}32770$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47308$

$\log \beta = 5{\cdot}60784$

(Table IV.) $\log r^3 = 0{\cdot}00042$

$\log \frac{1}{\delta} = 9{\cdot}05611$

$9{\cdot}89787$

$m\,b\,\beta \frac{r^3}{\delta} = 0{\cdot}79044$

$$m\,b\,\beta \frac{R^3}{\Delta} - m\,b\,\beta \frac{r^3}{\delta} = 0{\cdot}32770 - 0{\cdot}79044 = -0{\cdot}46274$$

$m = 5759{\cdot}99143$

$M = 5759{\cdot}52869$ nearly.

The logarithm of M is 3·76039, three unities in the fifth decimal less than log m. We obtain by using, as before, log M instead of log m,

$\log M\,b\,\beta \frac{R^3}{\Delta} = 9{\cdot}51545$ $\quad M\,b\,\beta \frac{R^3}{\Delta} = 0{\cdot}32768$

$$M\,b\,\beta \frac{R^3}{\Delta} - m\,b\,\beta \frac{r^3}{\delta} = 0{\cdot}32768 - 0{\cdot}79044 = -0{\cdot}46276$$

$m = 5759{\cdot}99143$

Correct value of $M = 5759{\cdot}52867$

Reduction of the weighings of S.b.

U supposed of Brass.

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47661$

(Table I.) $\log \alpha = 4{\cdot}70615$

$9{\cdot}94318$

$c = 0{\cdot}01220$

$9{\cdot}93098$

$m\,b\,\beta \frac{R^3}{\Delta} = 0{\cdot}85306$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47661$

$\log \alpha = 4{\cdot}70615$

$9{\cdot}94318$

$m\,b\,\alpha = 0{\cdot}87736$

$$m\,b\,\beta \frac{R^3}{\Delta} - m\,b\,\alpha = 0{\cdot}85306 - 0{\cdot}87736 = -0{\cdot}02430$$

$m = 5759{\cdot}98966$

$M = 5759{\cdot}96536$

The logarithm of M is 3·76042, the same as that of m, so that it is not necessary to repeat the calculation with log M.

U supposed of Copper.

$m\,b\,\beta \frac{R^3}{\Delta}$ as before $= 0{\cdot}85306$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47661$

(Table II.) $\log \beta = 5{\cdot}60880$

(Table IV.) $\log r^3 = 0{\cdot}00040$

$\log \frac{1}{\delta} = 9{\cdot}05611$

$9{\cdot}90234$

$m\,b\,\beta \frac{r^3}{\delta} = 0{\cdot}79862$

$$m\,b\,\beta \frac{R^3}{\Delta} - m\,\beta \frac{r^3}{\delta} = 0{\cdot}85306 - 0{\cdot}79862 = +0{\cdot}05444$$

$m = 5759{\cdot}98966$

$M = 5760{\cdot}04410$

The logarithm of M is 3·76042, the same as that of m, so that it is not necessary to repeat the calculation with log M.

Reduction of the weighings of RS.

U supposed of Brass.

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47430$

(Tab.II.) $\log\beta = 5{\cdot}60775$

(Tab.V.) $\log R^3 = 0{\cdot}00022$

$\log\frac{1}{\Delta} = 8{\cdot}67392$

$9{\cdot}51661$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47430$

(Tab. I.) $\log a = 4{\cdot}70512$

$9{\cdot}93984$

$m\,b\,a = 0{\cdot}87064$

$$m\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}32856$$

$$m\,b\,\beta\frac{R^3}{\Delta} - m\,b\,a = 0{\cdot}32856 - 0{\cdot}87064 = -\,0{\cdot}54208$$

$$m = 5759{\cdot}99795$$

$$M = 5759{\cdot}45587 \text{ nearly.}$$

The logarithm of M is 3·76038, four unities in the fifth decimal less than log m. We obtain by putting, as before, log M in the place of log m,

$$\log M\,b\,\beta\frac{R^3}{\Delta} = 9{\cdot}51657 \qquad M\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}32852$$

$$M b\beta\frac{R^3}{\Delta} - m b\beta\frac{r^3}{\delta} = 0{\cdot}32852 - 0{\cdot}87064 = -\,0{\cdot}54212$$

$$m = 5759{\cdot}99795$$

Correct value of $M = 5759{\cdot}45583$

U supposed of Copper.

$m\,b\,\beta\frac{R^3}{\Delta}$ as before $= 0{\cdot}32856$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47430$

(Table II.) $\log\beta = 5{\cdot}60775$

(Table IV.) $\log r^3 = 0{\cdot}00042$

$\log\frac{1}{\delta} = 9{\cdot}05611$

$9{\cdot}89900$

$$m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}79251$$

$$m b\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}32856 - 0{\cdot}79251 = -\,0{\cdot}46395$$

$$m = 5759{\cdot}99795$$

$$M = 5759{\cdot}53400 \text{ nearly.}$$

The logarithm of M is 3·76039, three unities in the fifth decimal less than log m. We obtain by putting, as before, log M in the place of log m,

$$\log M\,b\,\beta\frac{R^3}{\Delta} = 9{\cdot}51658 \qquad M\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}32853$$

$$M b\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}32853 - 0{\cdot}79251 = -\,0{\cdot}46398$$

$$m = 5759{\cdot}99795$$

Correct value of $M = 5759{\cdot}53397$

Reduction of the weighings of RM.

U supposed of Brass.

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47245$

(Tab. I.) $\log a = 4{\cdot}70497$

$c = 0{\cdot}00033$

$9{\cdot}93817$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47245$

$\log a = 4{\cdot}70497$

$9{\cdot}93784$

$m\,b\,a = 0{\cdot}86664$

$$m\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}86730$$

$$m\,b\,\beta\frac{R^3}{\Delta} - m b a = 0{\cdot}86730 - 0{\cdot}86664 = +0{\cdot}00066$$

$$m = 5760{\cdot}00887$$

$$M = 5760{\cdot}00953$$

The logarithm of M is 3·76042, the same as that of m, so that it is not necessary to repeat the calculation with log M.

U supposed of Copper.

$m\,b\,\beta\frac{R^3}{\Delta}$ as before $= 0{\cdot}86730$

$\log m = 3{\cdot}76042$

$\log b = 1{\cdot}47245$

(Table II.) $\log\beta = 5{\cdot}60760$

(Table IV.) $\log r^3 = 0{\cdot}00042$

$\log\frac{1}{\delta} = 9{\cdot}05611$

$9{\cdot}89700$

$$m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}78886$$

$$m\,b\,\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}86730 - 0{\cdot}78886 = +0{\cdot}07844$$

$$m = 5760{\cdot}00887$$

$$M = 5760{\cdot}08731 \text{ nearly.}$$

The logarithm of M is 3·76043, by one unity in the fifth decimal greater than log m. We obtain by using, as before, log M instead of log m,

$$\log M\,b\,\beta\frac{R^3}{\Delta} = 9{\cdot}93818 \qquad M\,b\,\beta\frac{R^3}{\Delta} = 0{\cdot}86732$$

$$M b\beta\frac{R^3}{\Delta} - m\,b\,\beta\frac{r^3}{\delta} = 0{\cdot}86732 - 0{\cdot}78886 = +0{\cdot}07846$$

$$m = 5760{\cdot}00887$$

Correct value of $M = 5760{\cdot}08733$

34. We may now put, in one view, the results thus obtained, using only four decimals, which is enough for the accuracy, of which the operation of weighing is capable. I have only used five decimals in the reductions, in order to have the fourth decimal not affected by the calculation.

The reductions in § 33. give for the absolute weight of the five several pounds, compared with U, the following values, expressed in grains troy: the first when we suppose U of brass with the specific gravity = 8·0, the second when we suppose U of copper with the specific gravity = 8·788.

	U of Brass.	U of Copper.
Absolute weight of K	= 5760·0346	= 5760·1124
Absolute weight of S.[p]	= 5759·4507	= 5759·5287
Absolute weight of S.[b]	= 5759·9654	= 5760·0441
Absolute weight of RS	= 5759·4558	= 5759·5340
Absolute weight of RM	= 5760·0095	= 5760·0873

If now the lost imperial standard troy pound should be restored by these five pounds, it must be made,

if U was of Brass,		if U was of Copper,	
0·0346 gr.	lighter than K	0·1124 gr.	lighter than K
0·5493	heavier than S.[p]	0·4713	heavier than S.[p]
0·0346	heavier than S.[b]	0·0441	lighter than S.[b]
0·5442	heavier than RS	0·4660	heavier than RS
0·0095	lighter than RM	0·0873	lighter than M.

The uncertainty of course that remains about the absolute weight of the lost standard is, by comparison with

My brass pound, made by Bate, . . .	denoted by K	= 0·0778 gr. troy.
My platina pound, ...	denoted by S.[p]	= 0·0780
My brass pound, made by Robinson, .	denoted by S.[b]	= 0·0787
The Royal Society's platina pound, . .	denoted by RS	= 0·0782
The Royal Mint brass pound, ...	denoted by RM	= 0·0778

or nearly 0·08 grain by all of them.

35. Nor is this uncertainty brought within much smaller limits if we adhere to the most probable hypothesis, and suppose U of brass. The specific gravity of brass being a compound metal, varies very much according to its composition. I have known brass to vary from 7·9 to nearly 8·5 specific gravity*. In the same manner as before we may find the absolute weight of S.[p], if we assume seven different specific gravities for U from 7·9 to 8·5, proceeding by 0·1.

* I have recently met with a piece of cast brass (intended for a pendulum bar) the specific gravity of which is *less* than 7·4.—F. Baily.

We thus obtain for the absolute weight of S.[P], if U had its specific gravity,

= 7·9 .... 5759·4397 grains troy.
= 8·0 .... 5759·4507 ——
= 8·1 .... 5759·4614 ——
= 8·2 .... 5759·4719 ——
= 8·3 .... 5759·4821 ——
= 8·4 .... 5759·4921 ——
= 8·5 .... 5759·5018 ——

There remains consequently (if even we suppose U of brass) an uncertainty about its absolute weight = 5759·5018 − 5759·4397 = 0·0621 gr., or about 0·06 gr.

In fact, though we have five different pounds in excellent preservation, and compared with the lost standard with the greatest care and the best instruments, and though the number of these comparisons surpasses 600, there remains an uncertainty of 0·08 gr., or at least 0·06 gr., as to its real weight; and this solely on account of its specific gravity and expansion not being known. It is to be hoped that no pound will in future ever be declared a legal standard, unless these elements (the knowledge of which is indispensable even for a single comparison with a good balance) are previously determined with the greatest possible precision. A standard pound is intended for the purpose of obtaining from it accurate copies; and it therefore involves a contradiction if those elements are not well ascertained.

N.B. The formula, for the "Barometer reduction" in page 467, should have the sign − prefixed to it.